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Modelling Conditional Volatility in the Rand/ Dollar Exchange Rate

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1. INTRODUCTION

For the past few years the rand/ dollar exchange rate has been volatile to put it mildly. On January 2, 1997 R4.69 could buy you one US dollar. On December 20, 2001 the exchange rate was R13.00 to the dollar. The rand depreciated by almost 42 percent against the US dollar between September and December 2001. Since then the rand has appreciated. On February 18, 2004 the exchange rate stood on R6.56. This recent level of around R6.60 to the dollar was last seen in May 2000.

The volatility of the rand during 2001/2 and subsequent allegations of foul play led to the commissioning of Judge Myburgh to investigate the possible causes of the depreciation of the rand in 2001. The Myburgh Commission identified several macroeconomic factors that could have attributed to the depreciation. Among these were slower international economic growth, contagion from Argentina, the poor current account balance and the financial account's movement from a surplus to deficit. However, the Commission was unable to explain the acceleration of the depreciation during the last quarter of 2001.

A recent IMF working paper by Bhundai and Gottschalk (2003) investigated the sources of nominal exchange rate fluctuations in South Africa. They considered changes in relative output levels, prices and real exchange rates, domestic monetary policy and contagion as possible causes. Their conclusion was that financial market developments could possibly explain the depreciation of the rand, but that the exact causes remain unclear.

These inconclusive conclusions are not rare. A study by Cheung, Chinn and Pascual (2003) suggests that no empirical exchange rate model could survive the nineties. Some models do well at certain time horizons and for some exchange rates. But the same ones performing well within the estimation sample are not necessarily the best performers out-of-sample.

While most structural (long run) models seem to be unsuccessful in explaining real world exchange rate movements, the aim of this study is to try and explain the daily movements in the rand/ dollar exchange rate. This paper thus attempts to model the conditional volatility of the exchange rate. Although (G)ARCH models cannot forecast the exact exchange rate, its changing variance can be forecast.

2. RESEARCH METHOD

Heteroscedasticity (or conditional volatility) in the rand/ dollar exchange rate is modelled employing daily data. Different mean specifications are tested as well as variations of ARCH, GARCH, EGARCH and TARCH models. The regressions are also run over different time horizons.

3 PREVIOUS STUDIES

3.1 (G)ARCH studies

AutoRegressive Conditional Heteroscedasticity (ARCH) models rest on the presumption that forecasts of the future variance can be improved if these forecasts are conditioned on the current state of the variance. The basis of these models is found in the financial markets where periods of instability are followed by periods of relative stability. Forecasts that take into account the current information generally are more accurate than others. The ARCH model was first proposed by Engle (1982) and since then various types of this model has been proposed by others. Bollerslev (1986) proposed a more flexible lag structure by means of the GARCH(p,q) model while later EGARCH and TARCH models allows for nonlinearity - see Bollerslev et al (1992) for a more comprehensive description of these models. Since the introduction of these models, various studies applying them to financial data have appeared. The following section summarizes a few of these studies on exchange rates.

3.2 Studies on daily exchange rates

One of the earliest studies on daily exchange rates, was done by Hsieh (1988). He examined the statistical properties of daily rates of change of five exchange rates over the period 1974 - 1983. He found that exchange rate changes are not independent and identically distributed, that each day of the week has a different distribution and that means and variances change over time. In a later study he modelled the heteroscedasticity in these five exchange rates (Hsieh, 1989). He concluded that standard GARCH(1,1) and EGARCH(1,1) models are successful in removing the heteroscedasticity in the data. However, the EGARCH models fit the data better and do not indicate integrated variances, while the GARCH models do indicate integrated variances.

Diebold and Nerlove (1989) studied temporal volatility patterns in weekly data of seven nominal dollar spot exchange rates for the period 1973 – 1985. They concluded that the ARCH specification is an effective way of formalizing the observation that large changes tend to be followed by large changes and small by small. It is also consistent with the leptokurtosis in the exchange rate data and can be used to generate statistical and economically meaningful measures of exchange rate volatility.

McKenzie and Mitchell (1998) considered the ability of the Power ARCH model to capture the features of volatility in 17 daily spot exchange rates for the period 1986 – 1997. This class of ARCH model estimates the optimal power term, rather than imposing a squared power term on the data. They concluded that the sum of ARCH and GARCH coefficients is less than unity; and that shocks are transitory rather than permanent. Furthermore the simple GARCH(1,1) model proofed to be the preferable one while the power or leverage term did little to enhance the model.

Cilliers, Smit and Kotze (1996) did a study, similar to the ones mentioned above, on the daily movements of the rand/ dollar exchange rate between 1987 and 1992. Like all the other studies they employed the first difference of the log of the rates. The data was found to be leptokurtic with changing average and variance over time. Daily changes in the commercial rand was best estimated by a GARCH(1,1) model and the financial rand by a GARCH(1,2) model. The sum of the estimated GARCH parameters was close to one, suggesting an integrated process. They concluded that GARCH models lead to better forecasts than historical models.

4 EMPIRICAL STUDY

4.1 Statistical properties of the data

The data set for the empirical work includes daily data for the period January 2, 1997 until February 18, 2004. The time period was somewhat restricted by the availability of daily data on the S&P500 and HangSeng indices.

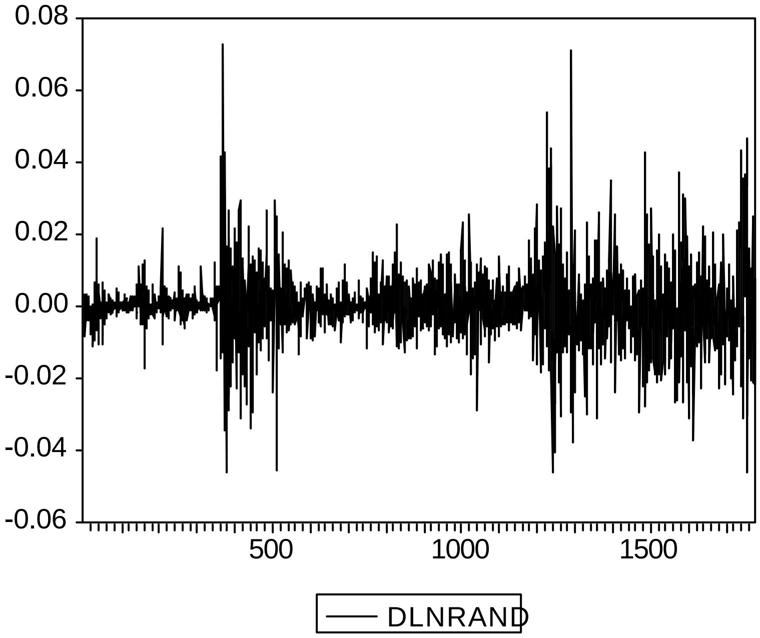
The variables are:

DLNRAND	First difference of the log of the rand/US dollar exchange rate
DLNSP500	First difference of the log of the closing value of the S&P500 index
DLNHS	First difference of the log of the closing value of the HangSeng index
DUMMA	Dummy variable taking a value of 1 for Mondays and 0 otherwise

The first difference of log was employed in order to render the data stationary, a precondition for the ARCH estimation technique.

The plot of DLNRAND in Figure 1 clearly indicates periods of high volatility (larger than usual variance) followed by periods of relative tranquility (smaller variance). This heteroscedasticity (not constant variance) is the focus of this study.

Figure 1: DLNRAND for the period January 1997 – February 2004



The summary statistics in Table 1 confirm some of the initial observations from Figure 1. At 0.000189 the mean value for the whole period is lower than the 0.000213 reported by Cilliers, Smit and Kotze (1996) as mean for the commercial rand during the period 1987 – 1992. There are, however, marked differences in the mean values of the respective years. Apart from the changing mean values, the volatility of the exchange rate is further confirmed by the standard deviations of especially 1998 and 2000 – 2004.

Table 1: Summary statistics of DLNRAND

Period	Obs	Mean	Standard deviation	Skewness	Kurtosis	JB	
1997.01 – 2004.02	1776	0.000189	0.010201	0.543	9.261	2988.0	*
1997	249	0.000150	0.003966	0.643	9.949	518.2	*
1998	249	0.000747	0.012700	0.724	8.570	343.4	*
1999	248	0.000195	0.006311	-0.868	16.638	1953.0	*
2000	246	0.000842	0.006515	0.237	2.782	2.8	
2001	250	0.001872	0.010480	0.856	8.669	365.2	*
2002	250	-0.001052	0.012259	0.269	3.676	7.8	
2003 – 2004	284	-0.000977	0.013545	0.297	4.035	16.9	*

* Reject H_0 of normality at a 1% level of significance?

The distributions of 2000 and 2002 are the closest to being normally distributed during the whole period. Their kurtosis values do not differ as much from 3 as the other periods. They are also the least positively skewed and therefore it is the only two years for which the null hypothesis of normal distribution cannot be rejected. However, the overall conclusion from Table 1 is that of a leptokurtic distribution, which is often observed in the financial markets.

Table 2: Statistical properties of DLNRAND

Period	DLNRAND stationary?	Autocorrelation in DLNRAND?	Autocorrelation in squared DLNRAND?
1997.01 – 2004.02	Yes	No	Yes
1997	Yes	Yes	Yes
1998	Yes	No	Yes
1999	Yes	Yes	Yes
2000	Yes	No	No
2001	Yes	No	Yes
2002	Yes	Yes	Yes
2003 – 2004	Yes	No	Yes

The statistical properties of the exchange rate variable are investigated further in Table 2. Corellograms and Ljung-Box Q-statistics (not quoted here) show no indication of autocorrelation, not for the whole period or any of the specific years individually. According to the Augmented Dickey-Fuller test, the time series is stationary over the whole period. Some indications of autocorrelation are found in three sub-periods, but for the whole period there is no significant autocorrelation present. The last column of Table 2 indicates the presence of autocorrelation in the squared value of DLNRAND. According to Hsieh (1989) this is indicative of strong heteroscedasticity.

4.2 GARCH models

In the light of the heteroscedasticity observed in the previous section and the successful empirical application of GARCH models to similar data, this section focuses on the modelling of the observed conditional volatility. Different specifications of the mean and variance equations are tested.

The mean equation is estimated as:

$$DLNRAND_t = a_0 + a_1 DLNRAND_{t-1} + a_2 DLNHS_t + a_3 DLNHS_{t-1} + a_4 DLNSP500_t + a_5 DLNSP500_{t-1} + ?_t \quad (1)$$

and the variance equation as:

$$h_t = \alpha_0 + \beta_1 \varepsilon^2_{t-1} + \beta_2 h_{t-1} + \beta_3 DUMMA_t + \beta_4 DLNHS_t + \beta_5 DLNSP500_t \quad (2)$$

DLNHS and DLNSP500 are included in an attempt to capture the daily effect of world markets on the South African exchange rate. The S&P500 serves as a proxy for world markets in general while the HangSeng represents the sentiments in the emerging markets. One day lags of both these measures are also included to account for the difference in time zones as well as the possibility that market reaction to news and events take place over a time period and not necessarily immediately. The Monday-dummy tests whether the variance of daily exchange rates are larger when trading spans a weekend. This "day of the week" effect is often observed in financial markets – see Hsieh (1988) for one of many examples.

Table 3: Regression results for 1997 – 2004 (dependent variable DLNRAND)

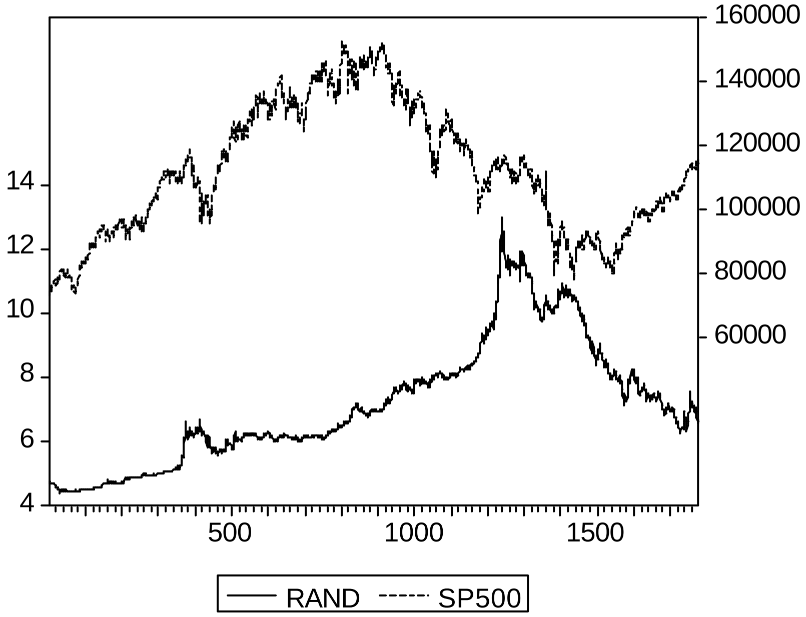
	1	2	3
Mean Equation			
C	0.000284**	0.000329***	0.000313***
DLNRAND(-1)	-0.044717#	-0.051323*	-0.046559#
DLNHS		-0.014529**	-0.018297***
DLNHS(-1)		-0.020094#	-0.014997*
Variance equation			
C			
ARCH(1)	0.175559***	0.169828***	0.249582***
GARCH(1)	0.841998***	0.846695***	0.734634***
DLNSP500			-0.000185***
DUMMA			-0.000004***
$\beta_1 + \beta_2$	1.017557	1.016523	0.984216

- All regressions are run with Bollerslev-Wooldrige robust standard errors and covariances
- Significance at 1% level indicated by ***
- Significance at 5% level indicated by **
- Significance at 10% level indicated by *
- Significance at 15% level indicated by #

Table 3 summarizes the results for regressions estimated over the whole period (January 1997 – February 2004). The regression results in the above table are the "best" ones both in terms of economic and statistical criteria. Experiments with GARCH(2,2) and GARCH(1,2) models did not provide better results. Asymmetric movements in the exchange rate were considered by estimating both TARCH and EGARCH models. These models account for the possibility that downward movements in the market are followed by higher volatilities than upward movements of the same magnitude. No signs of such asymmetry were found. The leverage term in the TARCH model, (RESID<0)*(ARCH(1)) was neither negative nor significant, while the leverage term in the EGARCH specification, RES/SQR[GARCH](1), was estimated positively instead of significantly negative.

The results in Table 3 emphasize a few interesting points. All the estimated parameters in the variance equation are statistically significant. However, in the mean equation at the top of the table, some are only significant at levels of 10 and 15%. This is in line with the findings of Hsieh (1989). He stated that few parameters in the mean equation have any statistical significance, while quite a number of parameters in the variance equation are significant. Changes in the HangSeng seem to affect changes in the exchange rate – with a stronger HangSeng leading to a stronger rand. Both the weekend and changes in the S&P500 affect the variance. The other alarming observation has to do with the sum of the ARCH and GARCH parameters. The first two regressions indicate an integrated process, where the persistence of volatility is high. However, the sum of the parameters of the third regression is less than one. Baillie and Bollerslev (1991) also reported much less persistence (sum far below 1) when including hourly dummies in their conditional variance equations of their study.

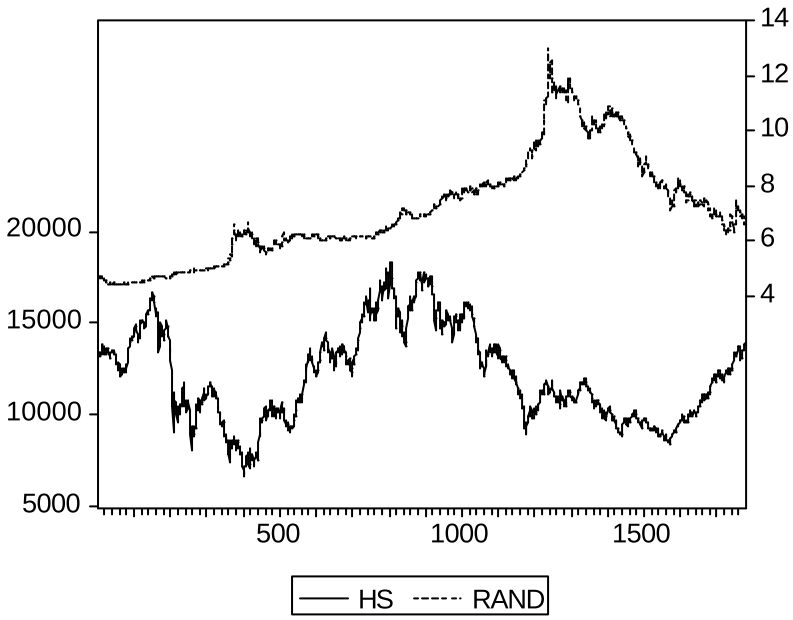
Figure 2: Relationship between exchange rate and S&P500



The summary statistics for DLNRAND show a very definite change in 2000 and again during 2002. Together with these changing properties within the data series itself, there also seems to be a change in the relationship between the exchange rate (RAND) and the S&P500 index. Figure 2 initially indicates that when US share prices increased, the dollar became more expensive (the Rand weakened). However, roughly since March 2000, a directly opposite relationship becomes evident. Higher US share prices were accompanied by a stronger rand.

In contrast with the above, the relationship between the exchange rate and the HangSeng index does not seem to have changed. Figure 3 indicates that a stronger rand was observed during periods when the HangSeng increased and the other way around. This means that improved economic conditions in the emerging markets context were reflected in a stronger rand.

Figure 3: Relationship between exchange rate and HangSeng.



To find out if/how the observed changes in the statistical properties of DLNRAND and the relationship between the exchange rate and S&P500 affect the estimated regressions reported in Table 3, separate regressions were run for two sub-periods. The first period is from January 1997 – February 2000 and the second from March 2000 – February 2004. The results are reported in Tables 4 and 5.

Table 4: Regression results for January 1997 – February 2000 (dependent variable DLNRAND)

	1	2	3
Mean Equation			
C	0.000367#	0.001174***	0.000485**
DLNRAND(-1)	-0.019756		
DLNHS	-0.054749***	-0.058768***	-0.061544***
DLNHS(-1)	0.018596**	0.018303	
Variance equation			
C		***	***
ARCH(1)	0.177089*	0.188195*	0.182723*
GARCH(1)	0.616655***	0.526187***	0.578024***
DLNSP500	-0.00159**	-0.000359	
DUMMA	0.00001**	-0.000052***	-0.000058***
$\beta_1 + \beta_2$	0.793744	0.714382	0.760747

The first column of Table 4 contains the results of the "best" specified regression estimated over the whole period. In this sub-period, however, the estimates are not significant. Regression 3 indicates that only a constant and changes in the HangSeng affects changes in the exchange rate. In the variance equation only the ARCH, GARCH and Monday-dummy are significant, while the S&P500 does not feature anymore. In the last row of Table 4 the sum of β_1 and β_2 is much lower than the ones reported in Table3. Regardless of the mean and variance specification, the GARCH process does not seem to be integrated.

Table 5: Regression results for March 2000 – February 2004 (dependent variable DLNRAND)

	1	2	3	4
Mean equation				
C	0.000153	0.000134	0.000232	0.000235
DLNRAND(-1)	-0.050898#	-0.048106#	-0.054323*	-0.053985*
DLNSP500		-0.026390#	-0.031902**	-0.031679*
DLNHS	0.010656			
DLNHS(-1)	-0.014784			
Variance equation				
C				***
ARCH(1)	0.180002***	0.200314***	0.110389***	0.089365***
GARCH(1)	0.833830***	0.798836***	0.889249***	0.910375***
DLNSP500	-0.000213*	-0.000161		
DLNHS			-0.000074**	-0.000093***
DUMMA	0.000004**	0.000002	-0.000005	
$\beta_1 + \beta_2$	1.013832	0.99915	0.999638	0.99974

The first regression reported in Table 5 again is a repetition of the "best" one reported in Table 3. And again, this specification does not render the same results as for the whole period. Since March 2000 the HangSeng does not play a role in explaining changes in the exchange rate. Its place is now filled by the S&P500. Almost the same happened in the variance equation. Previously changes in the S&P500 helped to explain changes in the variance, but now it is the HangSeng. The Monday-dummy also loses its explanatory power. The sum of β_1 and β_2 is very close to 1 and there are signs of an integrated GARCH process.

Two questions are raised to conclude. Does the mean equation matter? The relationship between the estimation of ARCH models and the specification of the mean equation has been researched by, amongst others, Gannon (1996). The general conclusion is that the specification of the mean equation bears little impact on the ARCH model. From the results reported in the first two columns of Tables 4 and 5, it does seem as if the mean equation matters. In both cases the changing mean equation render different results for the sum of β_1 and β_2.

The second question is: Does the variance equation matter? Yes. Regression 3 of Table 3 includes two extra variables not found in the variance equation of regression 2. The result is that β_1 and β_2 changes from indicating an integrated process (=1.0165) to no integration (=0.984). The inclusion of other variables, like the dummy for the Monday effect and proxies for other financial markets seems to lower the combined ARCH and GARCH coefficients.

5 SUMMARY AND CONCLUSION

The data representing changes in the log of the daily rand/ dollar exchange rate shows a leptokurtic distribution with changing mean and variance over time. The GARCH(1,1) model successfully models heteroscedasticity/ conditional volatility in the exchange rate. The inclusion of more explanatory variables in the mean and variance equations adds to the explanatory power of the estimated regressions. During the first part of the period the HangSeng features in the mean equation and the S&P500 in the variance equation. During the latter part, these variables swop places. The Monday-effect seems to affect the variance for the period as a whole and especially the first part. However, since 2000 others factors seem to be dominating. Lastly, there are some indications of an integrated variance, especially during the last part of the estimation period. This conclusion is sensitive for the specification of both the mean and variance equations and should be treated with the necessary caution.

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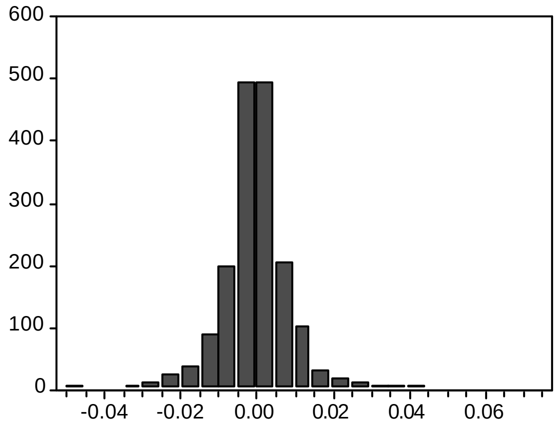
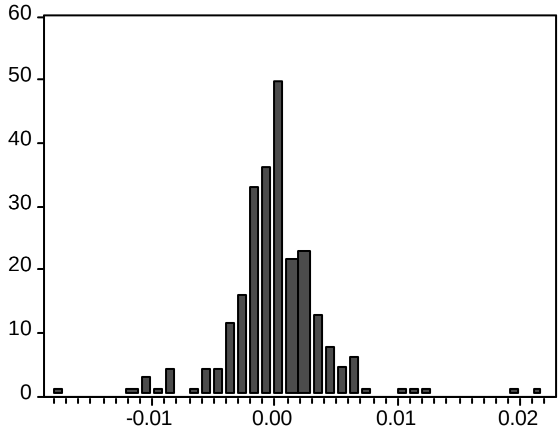
APPENDIX A

Whole period: Jan 1997 – Feb 2004



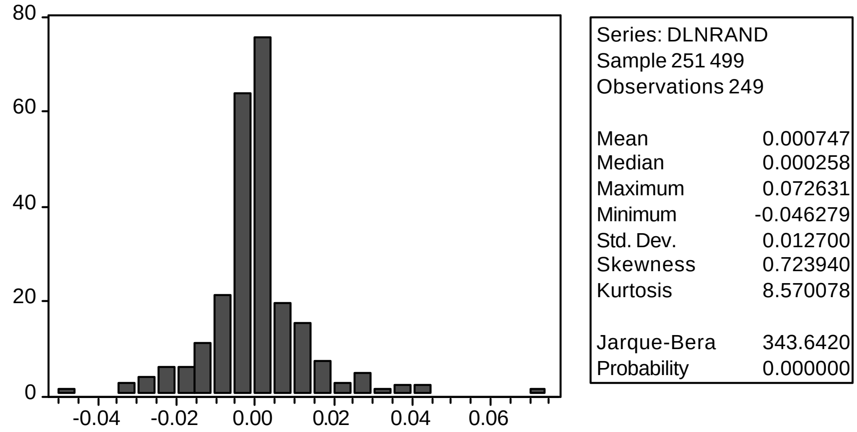
Series: DLNRAND	
Sample 2 1777	
Observations 1776	
Mean	0.000189
Median	0.000000
Maximum	0.072631
Minimum	-0.046366
Std. Dev.	0.010201
Skewness	0.542772
Kurtosis	9.261124
Jarque-Bera	2988.126
Probability	0.000000

1997

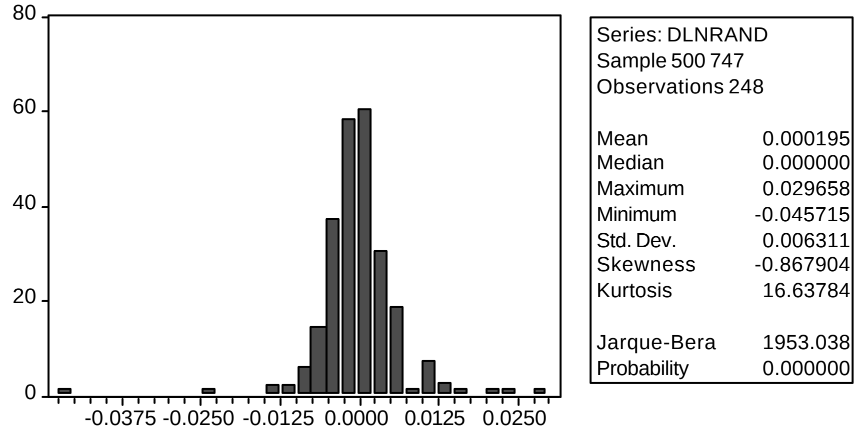


Series: DLNRAND	
Sample 2 250	
Observations 249	
Mean	0.000150
Median	0.000000
Maximum	0.021836
Minimum	-0.017073
Std. Dev.	0.003966
Skewness	0.643063
Kurtosis	9.948996
Jarque-Bera	518.1552
Probability	0.000000

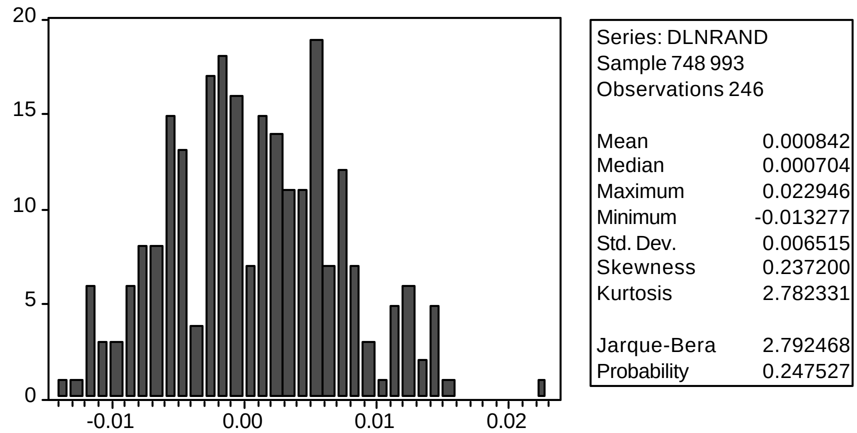
1998


Series: DLNRAND
Sample 251 499
Observations 249
Mean 0.000747
Median 0.000258
Maximum 0.072631
Minimum -0.046279
Std. Dev. 0.012700
Skewness 0.723940
Kurtosis 8.570078
Jarque-Bera 343.6420
Probability 0.000000
80
60
40
20
0
-0.04 -0.02 0.00 0.02 0.04 0.06

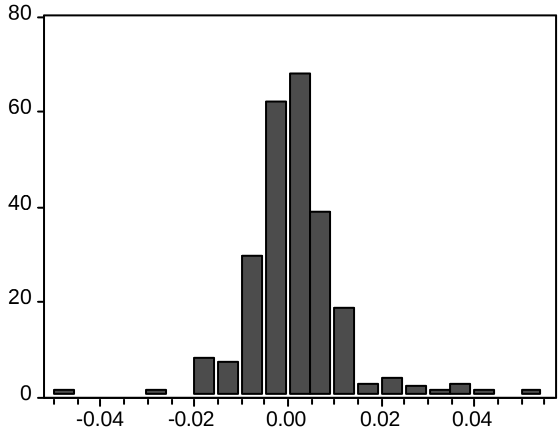
1999


Series: DLNRAND
Sample 500 747
Observations 248
Mean 0.000195
Median 0.000000
Maximum 0.029658
Minimum -0.045715
Std. Dev. 0.006311
Skewness -0.867904
Kurtosis 16.63784
Jarque-Bera 1953.038
Probability 0.000000
80
60
40
20
0
-0.0375 -0.0250 -0.0125 0.0000 0.0125 0.0250

2000

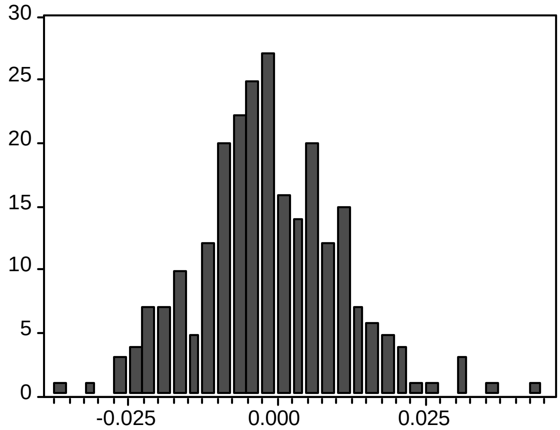

Series: DLNRAND
Sample 748 993
Observations 246
Mean 0.000842
Median 0.000704
Maximum 0.022946
Minimum -0.013277
Std. Dev. 0.006515
Skewness 0.237200
Kurtosis 2.782331
Jarque-Bera 2.792468
Probability 0.247527
20
15
10
5
0
-0.01 0.00 0.01 0.02

2001



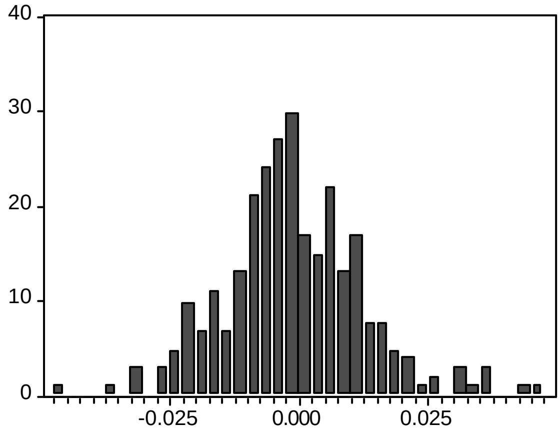
Series: DLNRAND	
Sample 994 1243	
Observations 250	
Mean	0.001872
Median	0.000924
Maximum	0.053619
Minimum	-0.046040
Std. Dev.	0.010480
Skewness	0.855963
Kurtosis	8.668589
Jarque-Bera	365.2457
Probability	0.000000

2002



Series: DLNRAND	
Sample 1494 1743	
Observations 250	
Mean	-0.001052
Median	-0.001628
Maximum	0.043416
Minimum	-0.037182
Std. Dev.	0.012259
Skewness	0.268694
Kurtosis	3.676459
Jarque-Bera	7.774805
Probability	0.020499

2003 – Feb 2004



Series: DLNRAND	
Sample 1494 1777	
Observations 284	
Mean	-0.000977
Median	-0.001736
Maximum	0.046533
Minimum	-0.046366
Std. Dev.	0.013545
Skewness	0.297492
Kurtosis	4.035062
Jarque-Bera	16.86674
Probability	0.000217