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Informational Asymmetry and Asset Pricing – A New Paradigm

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Abstract

I would like to sketch this article as one involving the game-theoretic framework that is expounded and used in the rest of the thesis. I intend to propose it as a mathematical foundation for probability. But it also has philosophical content; all the classical interpretations of probability fit into it. The framework begins with a two-person sequential game of perfect information. On each round, Player II states odds at which Player I may bet on what Player II will do next. In statistical modeling, Player I is a statistician and Player II is the world. In finance, Player I is an investor and Player II is a market. The framework is based on two principles: the principle of pricing by dynamic hedging (Player I can combine his bets over time), and the hypothesis of the impossibility of a gambling system[1]*, and the efficient market hypothesis*[2]*.*

[1] Cournots Equilibrium

[2] There is no strategy for Player I that can avoid all risk of bankruptcy and have a reasonable chance of making him rich.

The Rationale for Game Theoretic CAPM

Game theory describes the situations involving conflict in which the payoff is effected by the actions and counter-actions of intelligent opponents. Game theory and its classifications of non-cooperative behaviour could be extended to myriad applications of finance with an overwhelming extension into the annals of established capital asset pricing theories. Strategic interactions and its effective role in guiding effective investment decisions could be analysed from a game - theoretic framework.

An Investment Problem: Optimal Portfolio Selection

Consider the following investment problem. The problem is to decide what action or a combination of actions to take among three possible courses of action with the given rates of return as shown in the body of the following table.

		States of Nature (Events)			
		Growth	Medium G	No Change	Low
		G	MG	N	L
	Bonds	12	8	6	3
Actions	Stocks	15	7	3	-2
	Deposit	7	7	7	7

(All values are in percentage)

In decision analysis, the decision-maker has to select at least and at most one option from all possible options. This certainly limits its scope and its applications. Game theory concepts can be linked together with two seemingly different types of models (Decision theory and Linear Programming) to widen their scopes in solving more realistic decision-making problems. The investment problem can be formulated as if the investor is playing a game against nature.

Suppose our investor has \$100,000 to divide among the three possible investments. That is,

$$Y1 + Y2 + Y3 = 100,000$$

Notice that this condition is equivalent to the total probability condition for player I in the Game Theory.

Under these conditions, the returns are:

0.12Y1	+	0.15Y2	+	0.07Y3	{if Growth (G)}
0.08Y1	+	0.07Y2	+	0.07Y3	{if Medium G}
0.06Y1	+	0.03Y2	+	0.07Y3	{if No Change}
0.03Y1	-	0.02Y2	+	0.07Y3	{if Low}

The objective is that the smallest return (denoted by v value) be as large as possible.

Formulating this Decision Analysis problem as a Linear programming problem,

$$0.12Y1 + 0.15Y2 + 0.07\,Y3 \geq v$$

$$0.08Y1 + 0.07Y2 + 0.07Y3 \geq v$$

$$0.06Y1 + 0.03Y2 + 0.07Y3 \geq v$$

$$0.03Y1 - 0.02Y2 + 0.07Y3 \geq v$$

and Y1, Y2, Y3 ≥ 0, while v is unrestricted in sign (could have negative return).

In fact, the interpretation of this problem is that, in this situation, the investor is playing against nature (the states of economy). Solving this problem by any LP solution algorithm, the optimal solution is Y1 = 0, Y2 = 0, Y3 = 100,000, and v = \$7000. That is, the investor must put all the money in the saving money market account with the accumulated return of \$10,700. Note that the pay-off matrix for this problem has a saddlepoint[3], therefore, as expected, the optimal strategy is a pure strategy. In other words, one has to invest all his money into one portfolio only.

The Investment Problem Under Risk: Suppose the subjective probability for each of the states of economy is estimated to be: G(0.4), MG(0.2), NC(0.3), L(0.1). How should one invest to maximize the expected returns?

3

	G(0.4)		MG(0.2)		NC(0.3)		L(0.1)		**Expected value**
B	0.4(12)	+	0.2(8)	+	0.3(6)	+	0.1(3)	=	8.5*
S	0.4(15)	+	0.2(7)	+	0.3(3)	+	0.1(-2)	=	8.1
D	0.4(7)	+	0.2(7)	+	0.3(7)	+	0.1(7)	=	7

As one expects, one should invest all $100,000 in buying government bonds.

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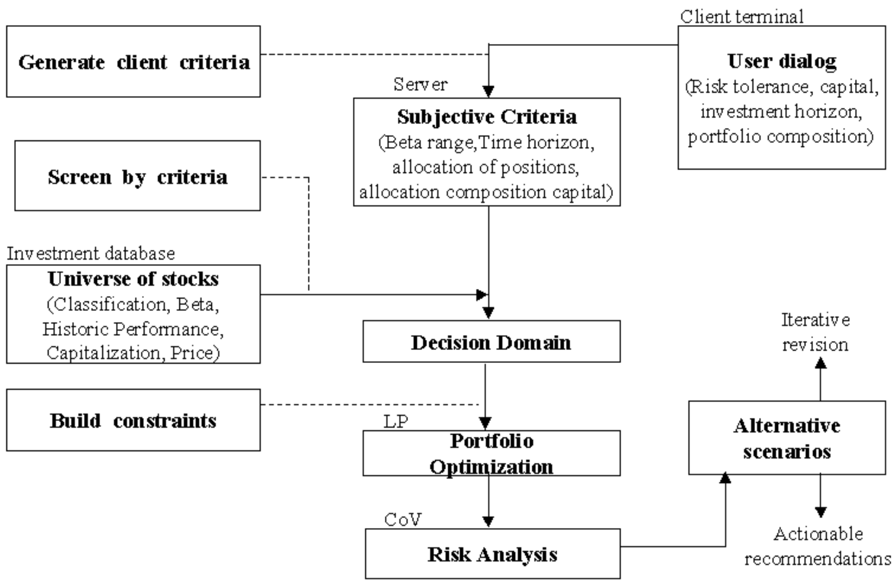
[3] Equilibrium Point



Risk Assessment Process: Clearly, different subjective probability models are plausible and they can give quite different answers. These examples show how important it is to be clear about the objectives of the modeling. An important application of subjective probability models is in modeling the effect of state-of-knowledge uncertainties in consequence models. Often it turns out that dependencies between uncertain factors can be important in driving the output of the models. For example, consider two portfolios having random variable R_1 and R_2 returns; the ratio:

$$Cov\ (R_1, R_2)\ /\ Var\ (R_1)$$

It is called the beta of the trading strategy 1 with respect to the trading strategy 2. Various methods are available to model these dependencies, in particular simple correlation methods. The following flowchart depicts the risk assessment process for portfolio selection based on their financial time series.

The above hybrid model brings together the techniques of decision analysis, linear programming, and statistical risk assessments (via a quadratic risk function defined by covariance matrix) to support the interactive decisions for modeling investment alternatives.

A Classification of Investors Relative Attitudes toward Risk and Its Impact

***Probability of an Event and the Impact of its Occurrence*:** The process-oriented approach of managing risk and uncertainty is part of any probabilistic modeling. It allows the decision-maker to examine the risk within its expected return, and identify the critical issues in assessing, limiting, and mitigating risk. This process involves both the qualitative and quantitative aspects of assessing the impact of risk.

Decision science does not describe what people actually do since there are difficulties with both computations of probability and the utility of an outcome. Decisions can also be affected by people's subjective rationality, and by the way in which a decision problem is perceived.

Traditionally, the expected value of random variables has been used as a major aid to quantify the amount of risk. However, the expected value is not necessarily a good measure alone by which to make decisions since it blurs the distinction between probability and severity. To demonstrate this, consider the following example:

Suppose that a person must make a choice between scenarios 1 and 2 below:

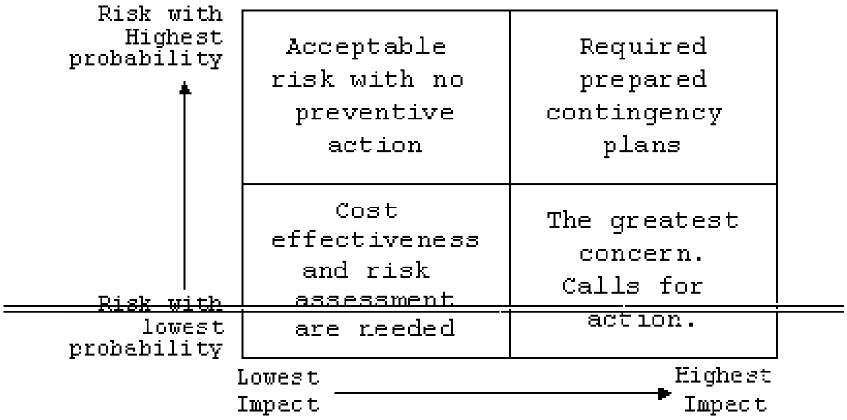
- Scenario 1: There is a 50% chance of a loss of $50, and a 50% chance of no loss.
- Scenario 2: There is a 1% chance of a loss of $2,500, and a 99% chance of no loss.

Both scenarios result in an expected loss of $25, but this does not reflect the fact that the second scenario might be much more risky than the first. (Of course, this is a subjective assessment).

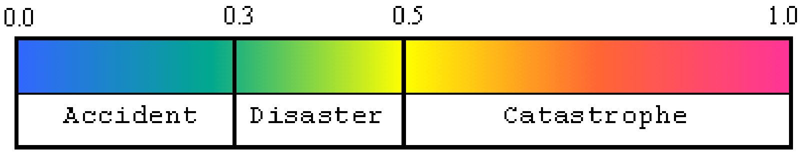
The decision-maker may be more concerned about minimizing the effect of the occurrence of an extreme[4] event than he/she is concerned about the mean.

The following charts depict the complexity of the probability of an event, the impact of the occurrence of the event, and its related risk indicator, respectively:

[4] Variance



Indicators for Subjective Behavior Toward Risk



Example of Risk Indicator

From the previous section, it may be recalled that the certainty equivalent is the risk free payoff; moreover, the difference between a decision-maker's certainty equivalent and the expected monetary value (EMV) is called the risk premium. One may use the sign and the magnitude of the risk premium in classifying a decision-maker's relative attitude toward risk as follows:

- If the risk premium is positive, then the decision-maker is willing to take the risk, and the decision-maker is said to be a risk seeker. Clearly, some people are more risk-seeker than others: the larger is the risk premium, the more risk seeker is the decision-maker.

- If the risk premium is negative, then the decision-maker would avoid taking the risk, and the decision-maker is said to be risk averse.

- If the risk premium is zero, then the decision-maker is said to be risk neutral.

Risk Assessment: An Objective Analysis to a Good Portfolio

Risk is the downside of a gamble, which is described in terms of probability. Risk assessment is a procedure of quantifying the loss or gain values and supplying them with proper values of probabilities. In other words, risk assessment means constructing the random variable that describes the risk. Risk indicator is a quantity that describes the quality of the decision.

Without loss of generality, consider the earlier Investment Example. Suppose the optimal portfolio is:

$$Y1.B + Y2.S + Y3.D$$

The expected value (i.e., the averages): The expected return is

$$Y1.B_r + Y2.S_r + Y3.D_r$$

where, B_r, S_r, and D_r are the historical averages for B, S, and D, respectively.

Expected return alone is not a good indication of a quality decision. The variance must be known so that an educated decision may be made. Have you ever heard the dilemma of the six-foot tall statistician who drowned in a stream that had an average depth of three feet?

In the investment example, it is also necessary to compute the 'risk' associated with the optimal portfolio. A measure of risk is generally reported by variance, or its square root called standard deviation. Variance or standard deviations are numerical values that indicate the variability inherent to one’s decision. For risk, smaller values indicate that what one expects is likely to be what he gets. What one desires is a large expected return, with small risk; thus, high risk makes the investor very worried.

Variance: An important measure of risk is variance:

$$Y1^2.Var(B)+Y2^2.Var(S)+Y3^2.Var(D)+2Y1.Y2.Cov(B,S)+2Y1.Y3.Cov(B, D) + 2Y2.Y3.Cov(S, D),$$

where Var and Cov are the variance and covariance, respectively, they are computed using recent historical data.

The variance is a measure of risk; therefore, the greater the variance, the higher the risk. The variance is not expressed in the same manner as the expected value. So, the variance is hard to understand and explain as a result of the squared term in its computation. This can be alleviated by working with the square root of the variance, which is called the **Standard Deviation**:

Both variance and standard deviation provide the same information and, therefore, one can always be obtained from the other. In other words, the process of computing standard deviation always involves computing the variance. Since standard deviation is the square root of the variance, it is always expressed in the same manner as the expected value. For the dynamic decision process, the Volatility as a measure for risk includes the time period over which the standard deviation is computed. The **Volatility measure** is defined as standard deviation divided by the square root of the time duration.

When considering two different portfolios, what does one do if his portfolio has a larger expected return but a much higher risk than the alternative portfolio? In such cases, using another measure of risk known as the **Coefficient of Variation** is appropriate.

Coefficient of Variation (CV) is the *absolute relative deviation* with respect to size provided it is not zero, expressed in percentage:

CV =100

Notice that the CV is independent from the expected value measurement. The coefficient of variation demonstrates the relationship between standard deviation and expected value, by expressing the risk as a percentage of the expected value. A portfolio with 10% or less CV is considered a "good" portfolio. The inverse of CV (namely 1/CV) is called the Signal-to-Noise Ratio.

Diversification May Reduce Risk: Since the covariance appears in risk assessment, it reduces the risk if it's negative. Therefore, diversifying ones investment may reduce the risk without reducing the benefits one gains from the activities. For example, one may choose to buy a variety of stocks rather than just one.

Game Theoretic CAPM

The established general theory of asset pricing (The CAPM or the capital asset pricing model stands out as the time tested theory in asset pricing) combines stochastic models returns with a rich tapestry of economic ideas: no arbitrage, general equilibrium, and marginal utilities for current and future consumption. (Campbell 2000; and Cochrane 2001). On the one hand they are hypotheses about the models of expected returns, which can be compared with the empirical distribution of returns and on the other hand they are hypotheses about investor's beliefs, which can be combined with the hypotheses about [5]investor's preferences to determine or predict asset prices. These two do not necessarily interact. Investors can be mistaken about the future and the fact that a stochastic[5] model predicted past prices is not a credible sign for investors to base it for their future decisions.

So there is the need for a greater clarity within this established theory, which consequently lists out certain parsimonious assumptions about the need to deconstruct stochasticity which means that a more than a modest assumption of the stochasticity of asset returns is required. So a more modest assumption could be generated in the game theoretic framework on capital asset pricing.

The framework necessarily includes a two-player perfect-information sequential game. On each round, Player I can buy uncertain payoffs at given prices, and then Player II determines the values of the payoffs. The game, a precise and purely mathematical object,

[5]A Random Process

is connected to the world by an auxiliary nonmathematical hypothesis, *Cournot's principle.* Cournot's principle says that if Player I avoids risking bankruptcy, then he cannot multiply his initial capital in the game by a large factor.

This principle gives empirical meaning to the game-theoretic forms of the classical limit theorems, for they say that certain approximations or convergences hold unless Player I is allowed to become very rich. The simplest form of the strong law of large numbers, for example, says that if there are in?nitely many rounds of play, and Player I is allowed to make an arbitrary even-money bet on a binary outcome (heads or tails) on each round, then he has a strategy that does not risk bankruptcy and makes him in?nitely rich if Player II does not make the proportion of heads converge to one-half.

A ?nancial market provides a game of the required form: Player I is a speculator, who may buy various securities at set prices at the beginning of each trading period, and Player II is the market, which determines the security's returns at the end of the period. If one measures Player I's capital relative to a particular market index, then Cournot's principle becomes an efficient market hypothesis:

Player I cannot beat the index by a large factor. In this chapter, an attempt is made to show that this ef?cient market hypothesis implies an approximate relation between an investor's actual returns and the index's actual returns.

The ef?cient market hypothesis is, of course, consistent with the established theory. In its in?nitary form, it can even be considered a consequence of the established theory. The established theory requires that the stochastic process for asset returns be absolutely continuous with respect to the risk-neutral probability measure obtained by normalizing state prices. Because asset prices are expected values for future returns with respect to the risk-neutral probability measure, this measure gives zero probability to the event that any given strategy for trading at these prices without risking bankruptcy will be in?nitely successful.

So the stochastic process must also give such an event zero probability. Thus the game-theoretic approach deconstructs rather than contradicts the established theory. It allows one to extract one part of the established theory—the efficient market hypothesis—and explore the consequences of this part alone. While not contradicting the established theory, the game-theoretic CAPM differs from it radically in spirit in the following way.

1. No assumptions are made whatsoever about the preferences or beliefs of investors.

2. Asset returns are not determined by a stochastic process. Instead these returns are determined by the market, a player in the game. The market may act as it pleases, except that it is constrained in a certain sense by the efficient market hypothesis—the expectation that it will not allow spectacular success for any particular investment strategy that does not risk bankruptcy.

3. The predictions of the model concern the relation between the actual returns of an investor (or the actual returns of a security or portfolio) and the actual returns of an index. These predictions are precise enough to be con?rmed or falsi?ed by the actual returns, without any further modeling assumptions.

The results lead to some startling conclusions: a new way of analyzing the past performance of portfolios and investors. According to the theory the underperformance of a portfolio can be approximated to one half of the empirical variance between the difference of the returns from the portfolio and that from the index, henceforth referred to as the *theoretical performance deficit.* In the case of an investor or fund whose strategy cannot be sold short because it is not public information, the theoretical performance de?cit should be a lower bound on the underperformance. Because a variance can be decomposed in many ways, the identification of the theoretical performance deficit opens the doors to a plethora of ways to analyse underperformance.

Average Return and Covariance

Consider a particular financial market and a particular market index “m” in which investors and speculators can trade. As per the Efficient Market Hypothesis (EMH) the speculator cannot beat “m” by a significant factor, which is the efficient factor hypothesis for “m”. The game-theoretic CAPM for “m” which follow from this hypothesis points out the fact that if any portfolio or trading strategy can be sold short, then its average simple returns, say $\boldsymbol{\mu_s}$, is approximated by

$$\mu_s \sim \mu_m - \sigma^2_m + \sigma_{sm} \qquad (1)$$

Where μ_s is the simple average return for the index, σ^2_m is the empirical variance for m’s simple returns and σ_{sm} is the covariance of s’s and m’s simple returns. This is referred to as the long short game- theoretic CAPM.

If the security cannot be sold short then,

$$\mu_s \leq \mu_m - \sigma^2_m + \sigma_{sm} \qquad (2)$$

This is also referred to as the long game-theoretic CAPM. It can also be written as

$$\mu \sim (\mu_m - \sigma^2_m) + \sigma^2_m \beta \qquad (3)$$

Where μ is written instead of μ_s in representing s’s average returns and β as σ_{sm}/σ^2_m . The line $\mu = (\mu_m - \sigma^2_m) + \sigma^2_m \beta$ in the (β,μ) plane is known as the security market line[6] for the game-theoretic CAPM. Here β is the sensitivity of s to

[6] Each point on the line represent combinations of investments, which yield the same level of, expected return and exhibits the same variance.

m and is the slope of the empirical regression passing through the origin of s's returns on m's returns.

The starting point of game-theoretic CAPM is the fact that the growth of the investment in s is best gauged not by its simple returns but by its logarithmic returns "In (1+ s)". Investing one unit in s at the beginning of each trading period, and reinvesting all dividends while proceeding through all the trading periods, the accumulated final wealth could be represented as Ws at the end of N trading periods, then

$$1/N \text{ In } W_s = 1/N \text{ In } ?_{n=1} (1+ Sn) = 1/N \sum_{n=1} \text{In } (1+Sn) \quad (4)$$

So the Taylor's expansion In (1+ x) ≈ x-1/2 x^2 yields,

$$1/N \text{ In } W_s = 1/N \sum_{n=1} (Sn - ½ Sn^2) \quad (5)$$

This is referred to as the "*fundamental approximation of asset pricing.*" Its shows that investors and speculators should be concerned about variance even if variance does not measure risk, for volatility diminishes the final wealth that one might expect from a simple average return. Moreover it establishes approximate indifference curves in the (s, µ) plane for a speculator bothered only about final wealth. These indifference curves can be reasoned in the same manner as the classical CAPM reasons an investors mean-variance indifference curves.

<u>RESEMBLANCE TO CLASSICAL CAPM</u>

Suppose one equates,

$$\mu_f = \mu_m - s^2_m \quad (6)$$

which can be rewritten in the form,

$$\mu_s = \mu_f + (\mu_m - \mu_f)\, s_{sm}/s^2_m \quad (7)$$

This resembles the classical CAPM, which can be rewritten as,

$$\mathbf{E(R_s) = R_f + (E\ (R_m)- R_f)\ Cov(R_s,R_m)/Var(R_m)} \qquad (8)$$

Where R_f is the risk-free return and $\mathbf{R_m}$ are random variables whose realizations are the simple returns $\mathbf{S_n}$ and $\mathbf{M_n}$. So the game-theoretic CAPM modifies the classical one in three fundamentally different ways.

1. It replaces theoretical expected values, variances and covariances with empirical quantities. (The game theoretic model has no probability measure and hence no theoretical quantities).

2. It replaces an exact equation between theoretical quantities with an approximate equation between empirical quantities, derived from the fundamental approximation and an efficient market hypothesis.

It replaces the risk free return with $\mathbf{\mu_m\text{-}s^2_m}$.

THEORETICAL PERFORMANCE DEFICIT

If Wm is the final wealth resulting from investment of one unit in the index m and Ws the final wealth of an investor who also begins with a single unit of capital then,

$$\mathbf{1/N\ In\ W_m\text{-}1/N\ W_s \text{»}^{FA}(\mu_m\text{-}1/2s^2_m)\text{-}(\mu_s\text{-}1/2s^2_s)\text{»}^{capm} 1/2s^2_s\text{-}s_{sm}}$$

$$\mathbf{+1/2s^2_m \quad = \quad 1/2s^2_{s\text{-}m}} \qquad (9)$$

Here FA indicates fundamental approximation

$$\mathbf{1/N\ In\ W = \mu\text{-}\ ½\ s^2} \qquad (10)$$

and CAPM indicates use of game-theoretic CAPM,

$$\mu_s - \mu_{m\ =}\ s_{sm} - s^2_m \qquad (11)$$

The final step uses the identity $s^2_{s\text{-}m} = s^2_s - 2s_{sm} + s^2_m$ where (s-m) is the vector of differences in the returns s-m = ($s_1 - m_1$, $s_N - m_N$)

So when an investor short sells a security,

$$1/N \, InW_m - 1/N InW_s \approx 1/2 s^2_{s\text{-}m} \qquad (12)$$

and when he cannot short sell a security,

$$1/N \, InW_m - 1/N InW_s \geq 1/2 s^2_{s\text{-}m} \qquad (13)$$

Thus s's average logarithmic returns can fall short of m's logarithmic returns by a factor equal to $s^2_{s\text{-}m}/2$, which is referred to as the Theoretical Performance Deficit.

If the market index is considered to be maximally diversified then s's theoretical performance deficit can be attributed to insufficient diversification. Decomposing the vector of simple returns s,

Its theoretical performance deficit can be separated as,

deficit due to non-unit sensitivity to m, $½ (ß - 1)^2 \; s^2_m$ (14)

deficit due to volatility orthogonal to m, $½ \; s^2_e$ (15)

These two parts of the deficit represent two different aspects of insufficient diversification. Many other decompositions are possible corresponding to events within and outside the market.

Such decompositions would be useful in analyzing the past performance of mutual funds, especially funds that do try to track the market. There is nothing in the theory to support the assumption that the theoretical performance deficit of a particular security or a portfolio need to persist from one period to another. A persistence that is highly predictable or substantial would enable the speculator to short the portfolio or security and beat the market, thereby contradicting the efficient market hypothesis.

In the case of a fund or a portfolio that cannot be shorted because of lack of public information, theoretical performance deficit or some components of the deficit may exist.

Quantifying The Efficient Market Hypothesis (EMH)

The EMH asserted here assumes that the speculator with limited capital means cannot beat the index by a significant factor. At first glance the above assumption would be viewed with great circumspection. No matter what market, what period of time and what index "m" chosen, one can retrospectively find strategies and even securities that beat the index by a substantial factor. So it may seem that these assumptions go against the spirit of the EMH hypothesis.

However what the EMH means is that it does not expect the speculators final wealth to exceed by a large factor the final wealth that he would have achieved by simply investing his initial wealth in the market index "m". If a is a number close to zero and if the speculator initially starts with initial wealth equal to a monetary unit, then the EMH strongly expects that his final wealth would be less than $1/a\ W_m$, where W_m is the final wealth obtained by him when he simply invests his initial wealth in "m" at the outset. When the market is expected to follow such behaviour; it means that the market will adopt a set of actions to ensure that his final wealth is less than $1/aW_m$.

BASIC CAPITAL ASSET PRICING GAME (CAPG)

The capital asset pricing game has two principal players, the speculator and the market. The speculator must decide on how much of each security to hold or short and the market decides on how the price of each security should evolve over periods of time. The third ally in the game is the investor who invests each day. The game is a perfect information one with each player knowing the others moves.

There are (K+1) securities in the market to be traded through N periods of time.

Predictions from the EMH

If the speculator is to win this game either he must become very rich relative to the market index "m" or the event A must happen. To formalize this idea the following formulation must hold: The EMH for "m" predicts the event A at level a if the speculator has a winning strategy in the basic CAPG with A as the auxiliary goal and a as the significance level.

However there is the lack of credibility in using this model to analyse empirical situations. The doubts would center on whether the players in the financial markets choose their strategies based on perfect information and whether there are only three categories of players. The game is related to an actual securities market where an investor may act prudently, he may choose a static strategy or a dynamic one or he may react to circumstances. Since the market and the investor play this game as a team against the speculator the market may be considered to represent all other institutions in the market.

The speculator in this game is an imaginary investor as referred to by the EMH. The hypothesis asserts that a speculator cannot multiply his initial capital by a substantial factor relative to the index "m". Conceptually all this relates to strategies and viewed in that manner the EMH does not expect any strategy selected in advance to beat m by a substantial factor.

The roles of the players in the game are as follows. The investor earns the simple returns s and the speculator creates a portfolio p wherein he mixes $\sum$ of s and 1- $\sum$ of m. The speculator can replicate the move of the investor or he may act on information or actions of the investor. The speculator mixes the investor's moves with "m" sometimes going short on the investors move to go long. Because these simple strategies are sufficient the EMH drawn to support the empirical statements would be weak. Instead of assuming that the speculator cannot beat the index by a large factor, it is enough to assume that the speculator cannot beat m by a large factor using strategies more complicated than those used by investors.

However some dubious assumptions would also have to be clarified as one moves along. All the players in the market are not as imaginative as the one in the game and do not observe the investors moves. So there is the obvious manifestation of not observing the actions of the imaginary speculator. It also sounds dubious when one imagines the market and the investor colluding as a team to play against the speculator.

All these circumspections vanish once the results of the EMH for "m" hypothesizes that a particular event A will occur at a particular significance level of a. The speculator has a winning strategy in this game for particular values of parameters. These results remain valid if one restricts the knowledge and freedom of action of the investor and the market, the speculator can win the game as described as well as any game in which opponents are weaker. Since the market and the investor are endowed with so much knowledge and freedom of action in the game, the speculator can achieve this goal no matter how strategically market plays, no matter how much the market and the investor knows and no matter how much the market and investor collude.

To sum it up, the game-theoretic CAPM in no way attempts to refute the established theory. It was rather an alternative method to view asset pricing techniques from a wider paradigm, viewing it as a strategic interaction game between the principal players in the market. The EMH is very weak as can be seen; it only say's that the speculator cannot beat the market by using some rather obvious strategies. The predictions of this game-theoretic CAPM are loose, on the sense that they give fairly large bounds on the relation between average return and covariance between the market. But these bounds are themselves quite precise and can therefore be tested with no auxiliary stochastic model for individual returns. This precision enables the game-theoretic version to compare favourably with the classical CAPM raising the question of whether the classical model's much stronger assumptions give it greater predictive power. The exposition used in the chapter is purely theoretic. However this paper has attempted to view the established principles from a new framework seeking in the process to diffuse the theoretical esotericism associated with asset pricing and to stimulate alternative thought processes.

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