



**NINTH ANNUAL CONFERENCE ON  
ECONOMETRIC MODELLING FOR AFRICA**

**30 June  
to  
2 July 2004**

**Prospect Theory: Does it Warrant a Paradigm Shift in the  
Economics of Risk?**

Powell L. Mohapi

# Prospect Theory: Does it Warrant a Paradigm Shift in the Economics of Risk? <sup>S</sup>

Powell L. Mohapi  
Department of Economics  
National University of Lesotho  
P.O. Roma 180  
Lesotho  
Tel: +266-2221-3594/85  
Fax: +266-2234-0000  
pl.mohapi@nul.ls / plmohapi@yahoo.co.uk

## Abstract

This essay assesses whether the time has come for a paradigm shift – from expected utility framework to prospect theory framework – in the economics of choice under risk. A brief overview of the subject is outlined, starting with expected utility theory and noting its descriptive limitations. A catalogue of theories that have been proposed to make up for these limitations is also provided. This essay, like many scholars in the field, recognizes prospect theory as the most serious challenger to expected utility theory. A review of some descriptive predictions of prospect theory, particularly where expected utility proved inaccurate, suggest that there is no scientific reason why expected utility should not be ousted from dominance by prospect theory.

**Keywords:** Cumulative Prospect Theory, Expected Utility Theory

**JEL Classification:** D81

---

\* I wish to acknowledge the mentorship of Horst Zank, my MSc supervisor during my time at Manchester, to whom I owe my understanding of the economics of choice under risk. I remain solely responsible for errors herein.

*“In a series of influential articles, Kahneman and Tversky (1979, 1986, 1992) suggest that economic agents do not maximize the expected value of Von Neumann-Morgenstern utility function, but behave according to a set of rules collectively known as prospect theory.”- Shumway (1997, p.1)*

## **I. Introduction**

The subject of this essay is underpinned by the ongoing debate in the economics profession, in particular the economics of choice under risk. The debate concerns the continued supremacy of the classical expected utility theory (EUT) in the light of a growing body of evidence, albeit experimental, that individuals do not maximize the expected utility (EU). In the light of this evidence a proliferation of theories, collectively known as non-EU theories, has been put forth in an attempt to provide a better descriptive picture of observed behavior under risk. The development of non-EU models has a daunting task of coming with a model that will oust the EUT from its dominant position. The ultimate influence of the non-EU models on the profession at large will depend on whether they can be used to conduct analyses of standard economic decisions under risk in a manner that can match or at least approximate the elegance and power of the EU hypothesis (Machina, 1989; Epstein, 1992).

The starting point to the field is the fact that decision-making under risk is viewed as choice among lotteries (or prospects or gambles). In the 17<sup>th</sup> century conventional wisdom in probability theory was that the attractiveness of a lottery was given by its expected value (Machina, 1987). This view was shown to be flawed by Nicholas Bernoulli's St. Petersburg gamble.<sup>1</sup> A solution to the St. Petersburg paradox was proposed by Daniel Bernoulli (1738/1954) with the observation that expected utilities of payoffs instead of expected values of payoffs are what carry the attractiveness of the gamble. Of importance was the

---

<sup>1</sup> See Machina (1987) or Schoemaker (1982) for description of the game. Alternatively see [http://plato.stanford.edu/archives/fall\\_2001/entries/paradoxstpetersburg](http://plato.stanford.edu/archives/fall_2001/entries/paradoxstpetersburg)

conjecture that payoffs have decreasing marginal utilities invoked in Bernoulli's explanation that ensured the finite expected utility for the St. Petersburg gamble. EUT, the dominant hypothesis of behaviour under risk, originates from Bernoulli's resolution of the St. Petersburg paradox.

Of the numerous non-EU theories proposed in the wake of EUT descriptive inaccuracies prospect theory (PT) seems to be the most promising. The question raised in this essay is whether PT warrants a paradigm shift in theoretical analysis of choice under risk. In section II the EUT is outlined along with its well known descriptive inaccuracies while section III provides a cursory presentation of some prominent non-EU theories. A relatively in-depth treatment of PT is a subject of section IV. Some predictions of PT are provided in section V with section VI concluding.

## II. The Expected Utility Theory

Despite its success however, Bernoulli's first statement of the EUT did not receive any attention from economists. This was because the theory assumed a cardinal measure of utility while the economics profession was moving towards ordinal utility (Starmer, 2000). It was not until von Neumann and Morgenstern (1944) showed that the EU hypothesis could be derived from a set of few appealing axioms, rendering it to be a rational decision criterion, that interest in the theory was revived (Schoemaker, 1982; Starmer, 2000). A heuristic specification of the theory is provided below.

Let  $\{x_1, x_2, \dots, x_n\}$  be a set of monetary prizes or outcomes (that is, prizes are real numbers) and define a lottery  $X = (x_1, p_1; x_2, p_2; \dots; x_n, p_n)$  as a finite probability distribution over outcomes with  $p_i \in [0, 1]$  and  $\sum_{i=1}^n p_i = 1$ . A lottery  $X$  yields outcome  $x_i$  with probability  $p_i$  for  $i = 1, 2, \dots, n$ . Denote by  $A$  the set of all prizes/outcomes and by  $L$  the set of all lotteries. Let  $\succeq$  be a preference relation defined over lotteries.  $\succeq$  ( $\succ$ ) denotes a weak (strong) preference

relation. For some lotteries  $X$  and  $Y$ ,  $X \succeq Y$  and  $Y \succeq X$  imply  $X \sim Y$ . Furthermore let  $\succ$  satisfy the following axioms:

- A1. *Completeness*: For any two lotteries  $X$  and  $Y$  either  $X \succ Y$ ,  $Y \succ X$  or  $X \sim Y$ .
- A2. *Transitivity*: For any three lotteries  $X$ ,  $Y$  and  $Z$ , if  $X \succ Y$  and  $Y \succ Z$  then  $X \succ Z$ .
- A3. *Archimedean axiom*: If lotteries  $X$ ,  $Y$  and  $Z$  are such that  $X \succ Y \succ Z$  then there exist some  $a, \beta \in (0,1)$  such that  $aX + (1-a)Z \succ Y$  and  $Y \succ \beta X + (1-\beta)Z$ .
- A4. *Independence axiom*: If  $X \succ Y$  then  $(X, a; Z, 1-a) \succ (Y, a; Z, 1-a)$  for all  $a \in (0,1)$ .

*Completeness* and *transitivity* are familiar from conventional micro-theory. The *Archimedean* axiom serves the purpose served by continuity in conventional micro-theory. It effectively states that for any three lotteries that are strictly preferred to each other and are preference ordered, there exist some compound lotteries that are convex combinations of the most preferred and the least preferred lotteries such that one is strictly preferred to the middle lottery and the middle lottery is strictly preferred to the other. The *independence* axiom states that preferences between lotteries  $X$  and  $Y$  remains unaffected if they are each combined in the same way with the third lottery  $Z$ .

Preferences satisfying A1-A4 can be represented by the utility function

$$U(X) = \sum_{i=1}^n p_i u(x_i) \quad (1)$$

Where  $U: L \rightarrow \mathfrak{R}$  is the utility function representing preferences over lotteries and  $u: A \rightarrow \mathfrak{R}$  is the utility function representing preferences over outcomes.  $U(\cdot)$  satisfying (1) is the famous von Neumann-Morgenstern (VNM) utility function. The VNM utility function thus can be represented as a mathematical expectation of utilities over outcomes hence the name expected utility.

The VNM axiomatic characterization of Bernoulli's cardinal specification rendered it ordinal and as a result very attractive to economists. Since risk is common in economics, EUT was able to lend itself as an elegant modelling device in situations involving risk. Even some cardinal features of Bernoulli utility measure were accepted by economists the case in point being risk aversion. In order to compare degrees of risk aversion between agents cardinal utility is needed.

VNM EUT has since become the dominant hypothesis in the mainstream economics of risk. Several fundamental areas of economics are based on EUT. Starmer (2000) points to two such areas: *game theory* and *the analysis of dynamic choice*. EUT ensures the existence of Nash equilibrium in game theory while in dynamic choice problems sequential choices are dynamically inconsistent without EU preferences (Starmer, 2000). *Classical finance theory* is also underpinned by EU preference structure. In fact Campbell, Lo and MacKinlay (1997, Ch.1, p.3) sum this up with the words:

What distinguishes financial economics is the central role that uncertainty<sup>2</sup> plays in both financial theory and its empirical implementation. The starting point of every financial model is the uncertainty facing investors, and the substance of every financial model is the impact of uncertainty on the behaviour of investors and ultimately on market prices.

Despite its usefulness and popularity in theoretical modeling EUT has systematically failed to accurately explain observed behavior. Some prominent inaccuracies of EUT are stated below.

---

<sup>2</sup> Uncertainty, in this essay, should generally be understood as quantified into objective probabilities. In this way it becomes synonymous with risk.

### *Descriptive Inaccuracies of EUT*

EUT can exhibit either risk averse, risk neutral or risk seeking preferences – only one profile at a time. The first doubt of EUT predictive ability in this regard was raised by Friedman and Savage (1948) who noted that the coexistence of insurance and gambling appeared anomalous to this theory.

There is an abundance of experimental evidence, which reflects violations of the EUT axioms, particularly the independence axiom. One of the earliest examples of systematic violations of the independence axiom of the EUT is the well-known Allais paradox (Allais, 1953). Allais problem involved choice of a preferred option from each of the following two pairs of lotteries:

$$A1 = (\$1m, 1.00) \quad \text{vs} \quad A2 = (\$5m, 0.10; \$1m, 0.89; \$0m, 0.01)$$

and

$$A3 = (\$1m, 0.11; \$0m, 0.89) \quad \text{vs} \quad A4 = (\$5m, 0.10; \$0m, 0.90)$$

EUT predicts that if  $A1 \succ A2$  (or  $A2 \succ A1$ ) in the first pair then  $A3 \succ A4$  (or  $A4 \succ A3$ ) in the second pair respectively. However Allais (1953) and later Morrison (1967) and Slovic and Tversky (1974) found that almost all of the subjects who preferred  $A1$  in the first bet chose  $A4$  in the second bet. The Allais paradox is a special case of a more general empirical pattern known as *the common consequence effect* noted among others by Moskowitz (1974), Slovic and Tversky (1974) and Kahneman and Tversky (1979). Summary descriptions of this phenomenon can be found in Starmer (2000) and Machina (1987) with the latter author providing some intuitive explanation (1987, pp.129-130).

A closely related phenomenon, also stemming from Allais (1953), is the so-called *common ratio effect*. The common ratio effect is the observation that the more risky of the two lotteries becomes relatively more attractive when the probability of winning is reduced by equal proportion in both lotteries (Prelec, 1998). It is observed in choices among pairs of

problems of the form:  $X = (x, p; 0, 1-p)$  and  $Y = (y, ?p; 0, 1-?p)$  where  $? \in (0,1)$  and  $y \succ x$ . If  $?$  is constant EUT implies that preferences should not depend on the value of  $p$ . A number of studies however (Kahneman and Tversky, 1979; Loomes and Sugden, 1987; and Starmer and Sugden, 1989) reveal a tendency for individuals to switch their choices from  $y$  to  $x$  as  $p$  falls. The common consequence and the common ratio effects are violations of the independence axiom of the EUT.

Another class of the EUT preference violations concerns the elicitation of choices. The first phenomenon in this class is the *preference reversal*. In the classic experiment of this phenomenon by Lichtenstein and Slovic (1971) subjects were required to perform two tasks. Firstly, the subjects were asked to choose between two lotteries, the **\$**-bet (which offered a small chance of winning a good prize) and the **P**-bet (which offered a large chance of winning a small prize). The second task required subjects to assign monetary values,  $M(\cdot)$ , to the two lotteries as if they were to sell them. The first task revealed  $\mathbf{P} \succ \mathbf{\$}$  while the second revealed  $M(\mathbf{\$}) > M(\mathbf{P})$ . The robustness of this phenomenon is confirmed in Lichtenstein and Slovic (1973) and Grether and Plott (1979). Tversky and Thaler (1990) provide a review of further evidence on preference reversals.

The second phenomenon is the *framing (context) effects*. According to this phenomenon, the way in which objects of choice are described has a dramatic influence on choice. One widely cited example of this is the ‘Asian disease’ example of Tversky and Kahneman (1981) in which identical choices are framed in terms of *lives saved* to one group of subjects and *lives lost* to another group. Another example by Hershey and Schoemaker (1980) used the gambling and insurance frames as follows:

- |                  |  |
|------------------|--|
| Gambling frame:  | 1a. A sure loss of \$10.   |
|                  | 1b. 1% chance of losing \$1,000.                                   |
| Insurance frame: | 2a. Pay an insurance premium of \$10.                              |
|                  | 2b. Remain exposed to the hazard of losing \$1,000 with 1% chance. |

Both contexts present the choice between  $u(w_0-10)$  and  $0.01u(w_0-1,000) + 0.99u(w_0)$  and EUT description invariance dictates that choices should be consistent. However 56 percent of 42 Hershey and Schoemaker subjects preferred a sure loss when the problem was presented in the gambling context versus 81 percent in the insurance context.

Another predictive error of EUT has been reported recently by Rabin (2000) and Rabin and Thaler (2001). Rabin (2000) provides a theorem which shows that the risk neutrality result for very small bets extends to modest bets as well. His calibration results show that if risk aversion is invoked in modest bets, as is often the case in the economics literature, then the implied risk aversion for large bets is unrealistically high. For instance an EUT maximizer who rejects a 50-50 bet losing \$100 and gaining \$110 will also reject a 50-50 bet of losing \$1,000 and gaining *any* amount of money.

### **III. A Catalogue of Non-Expected Utility Theories**

The descriptive limitations of the EUT led to a wave of research in which a number of theories collectively referred to as Non-Expected Utility Theories were proposed. These theories can be classified into those that generalize EUT, mainly by relaxing the notion of linear probabilities, on one hand and those that are alternatives to EUT, which mainly abandon the rationality paradigm upon which EUT is based on the other. It is from this hunt of the EUT alternative that would pass the ultimate test, which is near-accurate prediction of observed behavior, that PT emerged as the most promising model. A selection of Non-EUT models is briefly reviewed before presenting the current version of PT in relative depth.

The well known of Non-EUT models are PT (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992), Regret Theory (Bell, 1982; Loomes and Sudgen, 1982) and Rank-Dependent (Expected) Utility Theory (Quiggin, 1982). Regret Theory, Rank-Dependent

Utility Theory together with Disappointment Aversion Theory (Gul, 1991) are briefly outlined below.

### *Regret Theory*

The intuition of regret theory is that the utility on a realized outcome also depends on what could have been the outcome had choice been made differently. That is, accompanying the resulting outcome from a decision is a psychological experience of either regret or rejoicing (Loomes and Sugden, 1982). This is best illustrated by Bell's (1982) example of a farmer whose crop value (price per bushel) at harvest time two months away is uncertain. Suppose the price will either be \$3 or \$7 with equal probability. If the farmer is offered a sure \$5 upfront and accepts but it turns out that at harvest the crop price is \$7 then he will experience regret of having lost \$2. On the other hand if the price turns out to be \$3 then there will be rejoicing over the \$2 gain.

Regret theory has explained with success some of the paradoxes of EUT. In both Loomes and Sugden (1982) and Bell (1982) the classical paradox of coexistence of insurance and gambling (Friedman and Savage, 1948) is well predicted by regret theory. The former authors also show that the Allais' paradoxes (common consequence and common ratio effects) are well explained by regret theory while in another paper (Loomes and Sugden, 1983) they point to regret as the rationale for preference reversals.<sup>3</sup>

### *Disappointment Aversion Theory*

Disappointment aversion developed among others by Gul (1991) is based on the idea that peoples' expectations are formed endogenously for each gamble and that they experience disappointment (elation) if the outcome of the gamble is less than (greater than) the

---

<sup>3</sup> In a later study by Tversky, Slovic and Kahneman (1990), intransitivity of preferences, which underpins regret as the rationale for preference reversals, accounted for only 10 percent of preference reversals observed while the remaining 90 percent was accounted for by failure of procedure invariance. Tversky et. al. point to failure of procedure invariance as the major cause of preference reversals.

expected outcome. The theory is compactly captured by the value function proposed by Jia, Dyer and Butler (2001).

$$v(x_i) = x_i + \begin{cases} e(x_i - Ex_i) & \text{if } x_i \geq Ex_i \\ d(x_i - Ex_i) & \text{if } x_i < Ex_i \end{cases} \quad (2)$$

where  $Ex_i$  is the expected outcome *ex ante* while  $x_i$  is the realized outcome *ex post* and  $d > e > 0$  reflecting that disappointment from a shortfall loom larger than elation from an excess of a corresponding magnitude.

#### *Rank-Dependent Utility Theory*

Rank-Dependent Utility (RDU) first proposed by Quiggin (1982) is based on the intuition that the attention people pay to a particular outcome depends not only on the likelihood of the outcome being realized but also on how good or bad the outcome is as determined by its relative position of desirability with respect to other possible outcomes. The second intuition of RDU derives from the lessons of the Allais' paradoxes that people do not process probabilities linearly. RDU transforms cumulative probabilities of each outcome into decision weights. This weighting scheme gave RDU an edge over other models that used decision weights in that it satisfied the notion of stochastic dominance while decision weights based on transformation of simple probabilities did not satisfy this condition.

The fitting of RDU non-linear weighting scheme produces an inverted S-shaped curve, which implies that people pay much attention to extreme (best and worst) outcomes and relatively less attention to intermediate outcomes (Quiggin, 1982; Diecidue and Wakker, 2001). This feature of RDU successfully explains the coexistence of gambling and insurance. Segal (1988) used RDU to explain the probabilistic insurance puzzle first observed by Tversky and Kahneman (1979). Because of its mathematical soundness, intuitive appeal and predictive ability RDU has been popular among economists relative to other non-expected utility models. That is echoed in the words of Machina (1994, p. 1237)

who referred to it “...as the most natural and useful modification of the classical expected utility formula”.

#### **IV. Prospect Theory**

Kahneman and Tversky (1979) proposed PT as a positive theory of choice in the wake of EUT inaccuracies in explaining observed behavior. PT comes in two versions: the original PT (OPT) of 1979 and its 1992 generalization, cumulative PT (CPT) by the same authors. OPT explained the major violations of EUT in choices between prospects involving a small number of outcomes, two nonzero outcomes to be specific.

One feature of OPT was to posit that choice is a two-phase process. In the first phase – the editing phase – people engage in some heuristic simplification of prospects into coding them into gains and losses and other tasks such as combination, segregation and cancellation. In the second phase edited prospects are evaluated and then choice takes place. Kahneman and Tversky (1979) note that many preference anomalies like intransitivities, isolation effects and context effects result from the editing phase. The editing operations are carried out in no specific order. The order in which they are carried out is likely to depend on the structure of the prospect, its frame and the format of its display (Kahneman and Tversky, 1979). In this way the editing phase does not lend itself to formal modeling. Due to this as well as the spirit of parsimony Kahneman and Tversky (1979) had to assume that the original formulation of the prospect leaves no room for further editing.

One of the major weaknesses of EUT is the notion of linearity in probabilities, which OPT attempted to get around by weighting outcomes with decision weights instead of their probabilities. The decision weights were monotonic transformations of outcome probabilities. The weighting scheme of transforming probabilities has two problems.

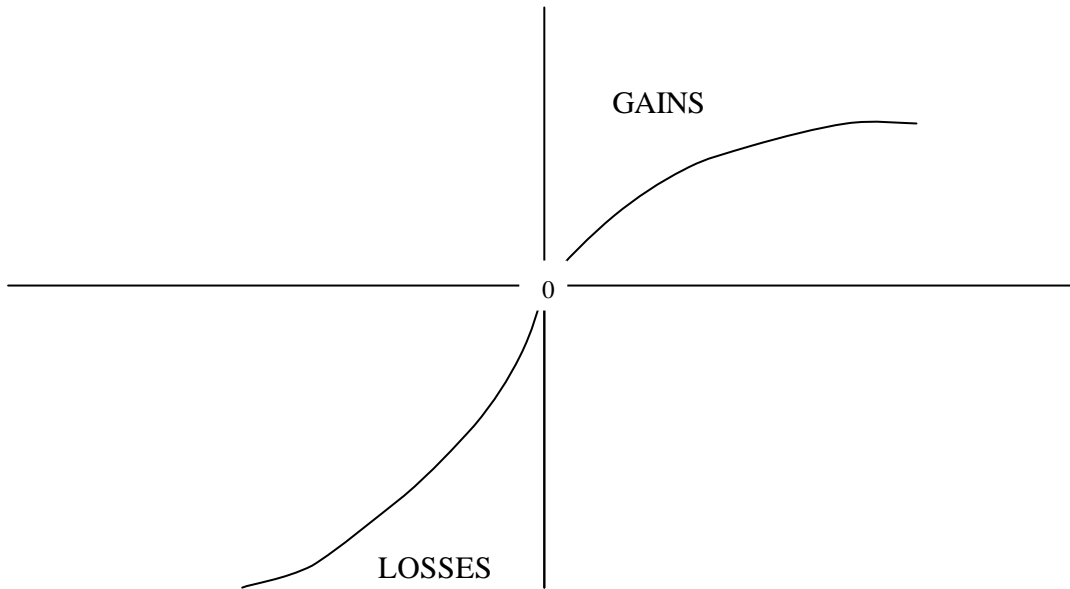
Firstly, as noted above, it could not always satisfy stochastic dominance and secondly it could not be readily extended to prospects involving a large number of outcomes.

### *Cumulative Prospect Theory*

To overcome the shortcomings of OPT Tversky and Kahneman (1992) put forth CPT. CPT embodies Quiggin's (1982) RDU while maintaining the basic features and intuition of OPT. The key features of CPT are:

- The carriers of value (analogue of utility) are changes in asset positions instead of final assets. The changes are viewed relative to some reference point or *status quo* and are perceived as gains and losses if they are positive and negative respectively.
- The value function has a kink at the reference point and is steeper for losses than for gains. It is also concave above the reference point while convex below the reference point. The first feature of the value function is implied by the fact that losses loom larger than corresponding gains. This is the notion of loss aversion (Kahneman and Tversky, 1979; Tversky and Kahneman, 1991). The second feature is a result of diminishing sensitivity, which means that the impact of a change in one's asset position diminishes with the distance from the reference point.
- CPT also allows for different treatment for gains and losses. Decision weights for gains and losses need not be the same. The value function is a two part functional, one for gains and the other for losses.

**Figure 1: The Prospective Value Function**



By embodying RDU, CPT inherits all the theoretical advantages of RDU that made it attractive to economists. However, CPT is more general than RDU by additionally allowing for reference- dependence and sign-dependence (Schmidt and Zank, 2002a).

### ***The CPT Model***

The model presentation in this section closely follows Tversky and Kahneman (1992) and Neilson and Stowe (2002).

Suppose a lottery  $X = (x_{-m}, p_{-m}; \dots; x_0, p_0; \dots; x_n, p_n)$  is composed of  $m + n + 1$  monetary outcomes which are rank ordered as  $x_{-m} < \dots < x_0 = \dots < x_n$ . Let  $x_0$  be the status quo and without loss of generality is set to zero (i.e.  $x_0 = 0$ ) so that outcomes  $\{x_i\}_{i=-m}^{-1}$  are losses and outcomes  $\{x_i\}_{i=1}^n$  are gains. The utility value of the lottery  $X$  under CPT is given by

$$V(X) = \sum_{i=-m}^n \mathbf{p}_i v(x_i) \quad (3)$$

Since CPT permits separate treatment of gains and losses  $V(X)$  can be decomposed into

$$V(X) = V^+(X) + V^-(X) \quad (4)^4$$

Where  $V^+$  measures the contribution of gains and  $V^-$  measures the contribution of losses.

Noting that  $v(x_0) = v(0) = 0$ , the terms in equation (3) can then be written as

$$V^+(X) = \sum_{i=0}^n \mathbf{p}_i^+ v(x_i) \quad \text{and} \quad V^-(X) = \sum_{i=-m}^{-1} \mathbf{p}_i^- v(x_i) \quad (5)$$

$\mathbf{p}_i$  denotes the decision weight for outcome  $x_i$  with  $\mathbf{p}_i^+$  ( $\mathbf{p}_i^-$ ) being a decision weight applied to a gain (loss).

As noted earlier the utility function for outcome  $x_i$ ,  $v(x_i)$  under CPT is a two-part functional to permit gains and losses to be borne differently. Tversky and Kahneman (1992) proposed the form:

$$v(x_i) = \begin{cases} x_i^a & \text{if } x_i \geq 0 \\ -\mathbf{I}(-x_i)^a & \text{if } x_i < 0 \end{cases} \quad (6)$$

The value function  $v(x_i)$  exhibits risk aversion over gains (by virtue of being concave above the reference point) and exhibits risk seeking over losses (since it is convex below the reference point). When faced with a choice between a sure gain and a prospect involving a probable loss and a probable gain, usually with a high upside risk, people tend to opt for a sure gain. On the other hand when faced with a choice between a sure loss and the prospect involving a probable gain and a probable loss, usually with a high downside risk, people tend to choose the prospect. These features have a loss aversion interpretation.

---

<sup>4</sup> The generality of CPT over RDU is clearly shown in this equation by observing that put another way (4) states that  $\text{CPT} = \text{RDU}^+ + \text{RDU}^-$ .

In the risk aversion case, an impossible loss (sure gain) is preferred to a probable loss while in the risk seeking case a probable loss is preferred to a sure loss.

The curvature properties of  $v(x_i)$  are ensured by the parameter  $a$  being restricted to be less than unity. The parameter  $\lambda$  is the coefficient of loss aversion and is strictly greater than unity. These parameters were estimated from an experimental study reported in Tversky and Kahneman (1992) to be  $a = 0.88$  and  $\lambda = 2.25$ . These estimates have since been used in subsequent numerous studies.

Several axiomatizations of CPT can be found in the literature. To derive the general functional form conditions complex beyond the standard properties (continuity, weak ordering, stochastic dominance) are often required. Luce and Fishburn (1991) use a condition termed compound gamble and joint receipt. Tversky and Kahneman (1992), Wakker and Tversky (1993), Chateauneuf and Wakker (1999) and Schmidt (2003) use sign-dependent comonotonic tradeoff consistency. The conditions can be less complex if a particular parametric form for utility is desired. Schmidt and Zank (2002b) use a condition called independence of common increments to derive a model where utility is linear for gains and linear for losses. Wakker and Zank (2002) use a generalization of constant proportional risk aversion to incorporate losses and derive CPT with utility as a power function. Zank (2001) provides a model where utility is exponential or linear by requiring constant absolute risk aversion for gains and losses separately.

#### *The Probability Weighting Function*

Decision weights are generated using a probability weighting function  $w(p)$ , in the following manner:

$$p_i^+ = \begin{cases} w^+(p_n) & \text{if } i = n \\ w^+\left(\sum_{k=0}^i p_{n-k}\right) - w^+\left(\sum_{k=0}^{i-1} p_{n-k}\right) & \text{if } i = \{0, \dots, n-1\} \end{cases}$$

(7)

$$p_i^- = \begin{cases} w^-(p_{-m}) & \text{if } i = -m \\ w^-\left(\sum_{k=0}^i p_{-(m-k)}\right) - w^-\left(\sum_{k=0}^{i-1} p_{-(m-k)}\right) & \text{if } i = \{m-1, \dots, -1\} \end{cases}$$

in which  $w^+(w^-)$  denotes a probability weighting function for gains (losses). The probability weighting function  $w(p)$  is strictly increasing and maps a unit interval into itself such that  $w(0) = 0$  and  $w(1) = 1$ . The parametric form of  $w(p)$  was proposed along with CPT by Tversky and Kahneman (1992) as:

$$w(p) = \frac{p^\beta}{(p^\beta + (1-p)^\beta)^{\frac{1}{\beta}}} \quad (8)$$

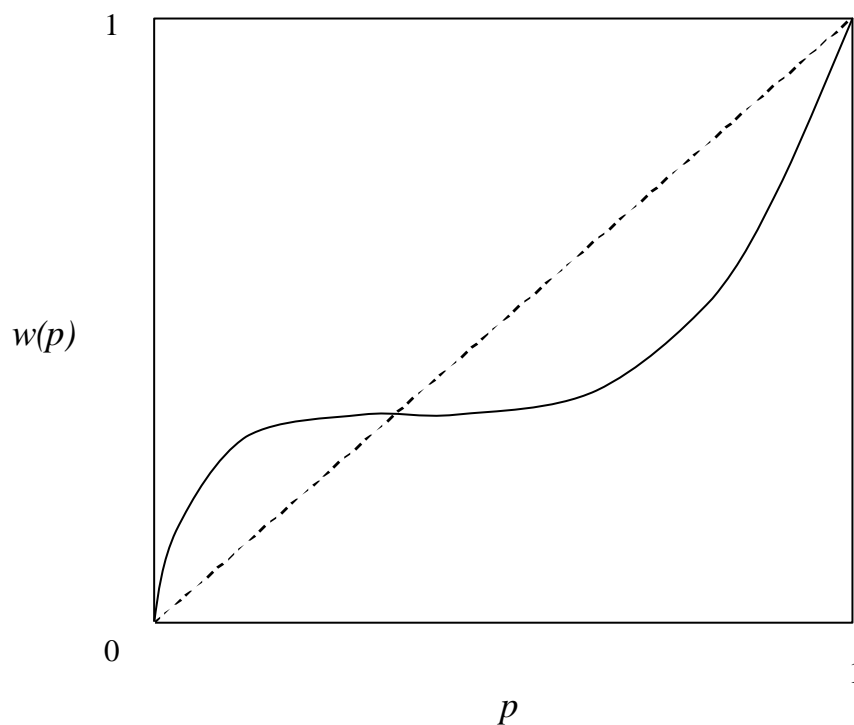
A fair amount of attention has been given to theoretical and empirical inquiry of the probability weighting function (Camerer and Ho, 1994; Gonzalez and Wu, 1999; Neilson and Stowe, 2002; Prelec, 1998; Tversky and Wakker, 1995; Wu and Gonzalez, 1996). All these studies converge to the conclusion that the probability weighting function has a shape of an inverted S. That is, it starts as concave and then becomes convex. There also seems to be a general agreement also that the point of inflection is below 0.5, in particular it is between 0.3 and 0.4. These general conclusions are robust to different parametric specifications.<sup>5</sup> In a quest to attenuate the dependence of the probability weighting function shape on the specific functional form used in parametric investigations Abdellaoui (2000) conducts a nonparametric inquiry into both the probability weighting function and the value (utility) function under CPT. Abdellaoui's (experimental) findings are in agreement with

---

<sup>5</sup> Gonzalez and Wu (1999) investigated  $w(p) = \frac{p^\beta}{p^\beta + (1-p)^\beta}$  while Prelec (1998) proposed and investigated  $w(p) = e^{-\ln p^\beta}$ .

findings obtained through parametric methods in other experimental studies, including those referred to herein.

**Figure 2: The shape of the Probability weighting Function**



The inverted-S shape of the probability weighting function implies that people overweight low probabilities and underweight high probabilities. This pattern suggests that people are overly sensitive to changes from impossible to possible and from possible to certain but are insufficiently sensitive to probabilistic information otherwise (Tversky and Wakker, 1995; Diecidue and Wakker, 2001). Inverted-S shape also predicts both optimism and pessimism. In an optimistic situation people tend to be risk seeking for gambles that yield gains with small probabilities such as is common with public lotteries. In a pessimistic situation people tend to be risk averse to gambles that yield losses with small probabilities, as is the case with insurance- (Diecidue and Wakker, 2001).

## **V. Prospect Theory Predictions**

The opening quote referred to PT as a “set of rules” due to the encompassing nature of the theory in terms of different and sometimes diverse situations that PT can explain with its many features or components that comprise the “set”, applied either individually or in groups. Embodied in CPT are the features of loss aversion, diminishing sensitivity, probability weighting and narrow framing. Loss aversion is arguably the central feature of CPT if the increased attention it has received in several areas of economic research is anything to go by. Many of the predictions reported below are prevalent in riskless choice.

Samuelson and Zeckhauser (1988) report a bias of preference towards an outcome that is designated as a status quo both from the field (Harvard faculty healthcare options) and from the laboratory. Johnson, Hershey, Meszaros and Kunreuther (1993) made a similar observation in a study of insurance purchases. Their study revealed that people have an exaggerated preference for whichever option is the default choice. A phenomenon of a slightly different sort is the endowment effect (Thaler, 1980) in which once a person is in possession of a good s/he exhibits strong reluctance to part ways with it even under appropriate incentives to do so (such as the offer to purchase the good or exchanging it for another good which is of greater value).

The three phenomena – status quo bias, default preference and the endowment effect – have a common feature of determining a reference point for the individual. The reference dependence nature of CPT helps to explain the phenomena with loss aversion and diminishing sensitivity. In the case of status quo bias and default preference the factor hindering change is that of diminishing sensitivity since the change to another option is very close to the reference point (status quo or default position). In the case of endowment effect loss aversion is the main factor responsible for the phenomenon. This is particularly so because the disutility of giving up a good is greater than the utility of acquiring it. These

arguments also help to understand the prevalence of brand loyalty to a particular product even when better products are available.

CPT also predicts asymmetries in the price elasticities of consumer goods due to loss aversion. Camerer (1998) reports results from studies by Putler as well as Hardie, Johnson and Fader to confirm this prediction. The catch here is that loss averse consumers dislike price increases more than they like price cuts of corresponding magnitudes. Consumers will cut back purchases more following a price increase compared to the extra amount they will buy following a price reduction.

The willingness to pay (WTP) to acquire a good and the willingness to accept compensation (WTA) to give up a good are the two measures of value in standard economic theory. While there is a disparity between these two measures due to, among other things income effects, they are often played down as not significant as shown by Knetsch and Sinden (1984, p.508) quoting other works positing that "...in most applications the error of approximation will be very small..." and "... practically speaking, it does not appear to make much difference which definition is adopted". The magnitude of the observed discrepancy is however large and is puzzling in standard models even after controlling for the suspected sources of disparity in an experimental setting (Knetsch and Sinden, 1984). Tversky and Kahneman (1991) invoke loss aversion to offer a compelling and plausible explanation for this discrepancy. The individual stating the WTA is the one in a position to give up a good while the person intending to acquire it declares WTP. If there is loss aversion for the good the owner will be reluctant to sell. Equivalently if the buyer views the money spent on the purchase as a loss there will be reluctance to buy. Kahneman, Knetsch and Thaler (1990) confirmed this behaviour experimentally.

The preceding section has already noted that the coexistence of insurance and gambling (state lotteries in particular though not exclusively), which could not be rationalised in the

standard framework, the EUT, follow directly from the CPT features of probability weighting and the simultaneous existence of concavity and convexity of the value function.

### *The Equity Premium Puzzle*

The high excess return on stocks was declared a puzzle by the work of Mehra and Prescott (1985). Since then a considerable amount of research effort has been directed at attempting to resolve the puzzle. The robustness of the puzzle to various resolution attempts is shown by Korchelakota (1996). However ideas from PT have rendered plausible explanations to the puzzle. Benartzi and Thaler (1995) use loss aversion coupled with narrow-framing to offer a convincing explanation of the phenomenon. The type of narrow-framing invoked is myopia (frequent evaluation periods of stocks performance). In an award winning article, Barberis, Huang and Santos (2001) couple PT with the house money effect to explain the equity premium puzzle. Their model is fully dynamic and consistent with rational expectations.

## **VI. Summary**

This essay has reviewed, briefly and heuristically, the economics of choice under risk with respect to whether a paradigm shift is necessary. The dominant behavioral hypothesis, the EUT, is plagued with descriptive limitations despite its normative appeal. Section III briefly reviewed some prominent non-EU models of individual decision making under risk after which PT was pointed to as the most serious challenger to the EUT. PT can explain most of the basic phenomena that EUT has been fruitfully applied to like asset pricing and insurance demand. In addition PT can explain behaviour that appears anomalous to EUT. After surveying ten such anomalies from the field, Camerer (1998) comments that there is no good scientific reason why EUT should not be abandoned in favour of PT and that PT should be given prominent space in economics textbooks.

Starmer's (2000) survey article suggests that the "hunt" is still on for the descriptive theory of choice under risk, which seems to imply that the non-EU theories that have been put forth have not yet matched the "elegance and power"<sup>6</sup> of EUT. The author here wishes to take a position: Let the hunt be called off and effort be directed towards refining and fine-tuning PT if that is necessary at all.

Some section of the profession feels that abandoning EUT would be tantamount to nullifying a lot of work that has been done in game theory, choice in dynamic settings (rational expectations models in particular) and to some extent finance. Machina's (1989) paper counters the widely held view that non-EU maximizers will behave in a manner that is dynamically inconsistent. He shows that dynamic consistency implied by the EUT is a result of an implicit assumption of *consequentialism*, which is a dynamic version of separability, which is rejected by non-EU maximizers. It is therefore not correct to impose consequentialism on non-EU preferences. Epstein (1992) surveys some non-EU theories that have been applied successfully to standard problems in macroeconomics, finance and game theory. EUT is therefore not indispensable.

The work of Tversky and Kahneman has recently, in 2002, been rewarded with a Nobel Prize in economics. This is a mark of the profession's awareness of their contribution to the economic science and if this is anything to go by, a paradigm shift may not be far off in the economics of risk.

\*\*\*\*\*

---

<sup>6</sup> The author notes that while EUT is hailed for its elegance and power in the rational paradigm, the behavioural approach has a completely different view. Leading behaviouralists Rabin and Thaler (2001) likened the recent criticism of EUT by Rabin (2000) to beating a *dead parrot* since the theory has been severely criticised for its descriptive inaccuracy shortly after its inception.

## References

- Abdellaoui, Mohammed (2000). "Parameter-Free Elicitation of Utility and Probability Weighting Functions," *Management Science* **46**, 1497-1512
- Allais, Maurice (1953). "Le Comportement de l'Homme Rationnel devant le Risque : Critique des Postulats et Axiomes de l'Ecole Americaine," *Econometrica* **21**, 503-542
- Barberis, Nicholas; Ming Huang and Tano Santos (2001). "Prospect Theory and Asset Prices," *Quarterly Journal of Economics* **116**, 1-53
- Bell, David E. (1982). "Regret in Decision Making under Uncertainty," *Operations Research* **30**, 961-981
- Benartzi, Shlomo and Richard H. Thaler (1995). "Myopic Loss Aversion and the Equity Premium Puzzle," *Quarterly Journal of Economics* **110**, 73-92
- Bernoulli, Daniel (1954). "Exposition of a New Theory on the Measurement of Risk," (original 1738), *Econometrica* **22**, 23-36
- Camerer, Colin F. (1998). "Prospect Theory in the Wild: Evidence from the Field," Social Science Working Paper #1037, California Institute of Technology
- Camerer, Colin F. and Tek-Hua Ho (1994). "Nonlinear Weighting of Probabilities and Violations of the Betweenness Axiom," *Journal of Risk and Uncertainty* **8**, 167-196
- Campbell, John Y.; Andrew W. Lo and A. Craig Mackinlay (1997). *The Econometrics of Financial Markets*, Princeton University Press
- Chateauneuf, Alain and Peter P. Wakker (1999). "An Axiomatization of Cumulative Prospect Theory for Decisions under Risk," *Journal of Risk and Uncertainty* **18**, 137-145
- Diecidue, Enrico and Peter P. Wakker (2001). "On the Intuition of Rank-Dependent Utility," *Journal of Risk and Uncertainty* **23**, 281-298
- Epstein, Larry G. (1992). "Behavior under Risk: Recent Developments in Theory and Applications," in Jean-Jacques Laffont (ed), *Advances in Economic Theory: Sixth World Congress Vol. II*. Cambridge University Press

- Friedman, Milton and Leonard J. Savage (1948). "The Utility Analysis of Choices Involving Risk," *Journal of Political Economy* **56**, 279-304
- Gonzalez, Richard and George Wu (1999). "On the Shape of the Probability Weighting Function," *Cognitive Psychology* **38**, 129-166
- Grether, David and Charles Plott (1979). "Economic Theory of Choice and the Preference Reversal Phenomenon," *American Economic Review* **69**, 623-638
- Gul, Faruk (1991). "A Theory of Disappointment Aversion," *Econometrica* **59**, 667-686
- Hershey, John C. and Paul H. Schoemaker (1980). "Risk-Taking and Problem Context in the Domain of Losses: An Expected Utility Analysis," *Journal of Risk and Insurance* **40**, 111-132
- Jia, Jianmin; James S. Dyer and John C. Butler (2001). "Generalized Disappointment Models," *Journal of Risk and Uncertainty* **22**, 59-78
- Johnson, Eric; Jack Hershey; Jacqueline Meszaros and Howard Kunreuther (1993). "Framing, Probability Distortions and Insurance Decisions," *Journal of Risk and Uncertainty* **7(1)**, 35-51
- Kahneman, Daniel and Amos Tversky (1974). "Judgement under Uncertainty: Heuristics and Biases," *Science* **185**, 1124-1131
- \_\_\_\_\_ and \_\_\_\_\_ (1979). "Prospect Theory: An analysis of Decision under Risk," *Econometrica* **47**, 263-291
- Kahneman, Daniel; Jack L. Knetsch and Richard H. Thaler (1990). "Experimental Tests of the Endowment Effect and the Coase Theorem," *Journal of Political Economy* **98**, 1325-1348
- Knetsch, Jack L. and J.A. Sinden (1984). "Willingness to Pay and Compensation Demanded: Experimental Evidence of an Unexpected Disparity in Measures of Value," *Quarterly Journal of Economics* **99**, 507-521
- Kocherlakota, Narayana R. (1996). "Equity Premium: It's Still a Puzzle," *Journal of Economic Literature* **34**, 42-71
- Levy, Haim; Enrico De Giorgi and Thorsten Hens (2003). "Prospect Theory and the CAPM: A Contradiction or Coexistence?" IERE working paper # 157, University of Zurich

- Lichtenstein, Sarah and Paul Slovic (1971). "Reversals of Preferences Between Bids and Choices in Gambling Decisions," *Journal of Experimental Psychology* **89**, 46-55
- \_\_\_\_\_ and \_\_\_\_\_ (1973). "Response-Induced Reversals of Preferences in Gambling: An Extended Replication in Las Vegas," *Journal of Experimental Psychology* **101**, 16-20
- Loomes, Graham and Robert Sugden (1982). "Regret Theory: An Alternative Theory of Rational Choice under Uncertainty," *Economic Journal* **92**, 805-824
- \_\_\_\_\_ and \_\_\_\_\_ (1983). "A Rationale for Preference Reversal," *American Economic Review* **73**, 428-432
- \_\_\_\_\_ and \_\_\_\_\_ (1987). "Some Implications of a More General Form of Regret Theory," *Journal of Economic Theory* **41**, 270-287
- Luce, Duncan and Peter Fishburn (1991). "Rank- and Sign-Dependent Linear Utility Models for Finite First Order Gambles," *Journal of Risk and Uncertainty* **4**, 29-59
- Machina, Mark J. (1987). "Choice under Uncertainty: Problems Solved and Unsolved," *Journal of Economic Perspectives* **1**, 121-154
- \_\_\_\_\_ (1989). "Dynamic Consistency and Non-Expected Utility Models of Choice under Uncertainty," *Journal of Economic Literature* **27**, 1622-1688
- \_\_\_\_\_ (1994). "Review of Generalized Expected Utility Theory: The Rank-Dependent Model by John Quiggin (1993) Kluwer Academic Publishers," *Journal of Economic Literature* **32**, 1237-1238
- Mehra, Rajnish and Edward C. Prescott (1985). "The Equity Premium: A Puzzle," *Journal of Monetary Economics* **15**, 145-161
- Morrison, Donald (1967). "On the Consistency of Preferences in Allais' Paradox," *Behavioral Science* **12**, 373-383
- Moskowitz, Herbert (1974). "Effects of Problem Presentation and Feedback on Rational Behavior in Allais and Morlat-Type Problems," *Decision Sciences* **5**, 225-242
- Neilson, William and Jill Stowe (2002). "A Further Examination of Cumulative Prospect Theory Parameterizations," *Journal of Risk and Uncertainty* **24**, 31-46
- Prelec, Drazen (1998). "The Probability Weighting Function," *Econometrica* **66**, 497-527

- Quiggin, John (1982). "A Theory of Anticipated Utility," *Journal of Economic Behaviour and Organization* **3**, 323-343
- Rabin, Matthew (2000). "Risk Aversion and Expected Utility Theory: A Calibration Theorem," *Econometrica* **68**, 1281-1292
- Rabin, Matthew and Richard H. Thaler (2001). "Anomalies: Risk Aversion," *Journal of Economic Perspectives* **15**, 219-232
- Samuelson, Paul A. (1963). "Risk and Uncertainty: A Fallacy of Large Numbers," *Scientia* **98**, 108-113
- Samuelson, William F. and Richard J. Zeckhauser (1988). "Status Quo Bias in Decision Making," *Journal of Risk and Uncertainty* **1**, 7-59
- Schmidt, Ulrich (2003). "Reference-Dependence in Cumulative Prospect Theory," *Journal of Mathematical Psychology* **47**, 122-131
- Schmidt, Ulrich and Horst Zank (2002a). "Risk Aversion in Cumulative Prospect Theory," mimeo
- \_\_\_\_\_ and \_\_\_\_\_ (2002b). "An Axiomatization of Linear Cumulative Prospect Theory with Applications to Portfolio Selection and Insurance Demand," mimeo
- Schoemaker, Paul H. (1982). "The Expected Utility Model: Its Variants, Purposes, Evidence and Limitations," *Journal of Economic Literature* **20(2)**, 529-563
- Segal, Uzi (1988). "Probabilistic Insurance and Anticipated Utility," *Journal of Risk and Insurance* **55**, 287-297
- Shumway, Tyler (1997). "Explaining Returns with Loss Aversion," University of Michigan Business School working paper
- Slovic, Paul and Amos Tversky (1974). "Who Accepts Savage's Axiom?" *Behavioral Science* **19**, 368-373
- Starmer, Chris (2000). "Developments in the Non-Expected Utility Theory: The Hunt for a Descriptive Theory of Choice under Risk," *Journal of Economic Literature* **38**, 332-382
- Starmer, Chris and Robert Sugden (1989). "Violations of the Independence Axiom in Common Ratio Problems: An Experimental Test of Some Competing Hypotheses," *Annals of Operational Research* **19**, 79-102

- Thaler, Richard H. and Eric J. Johnson (1990). "Gambling with the House Money and Trying to Break Even: The Effects of Prior Outcomes on Risky Choice," *Management Science* **36**, 643-660
- Thaler, Richard H.; Amos Tversky; Daniel Kahneman and Alan Schwartz (1997). "The Effect of Myopia and Loss Aversion on Risk Taking: An Experimental Test," *Quarterly Journal of Economics* **112**, 647-661
- Tversky, Amos and Daniel Kahneman (1981). "The Framing of Decisions and the Psychology of Choice," *Science* **211**, 453-458
- \_\_\_\_\_ and \_\_\_\_\_ (1986). "Rational Choice and the Framing of Decisions," *Journal of Business* **59**, s251-s278
- \_\_\_\_\_ and \_\_\_\_\_ (1991). "Loss Aversion in Riskless Choice: A Reference-Dependent Model," *Quarterly Journal of Economics* **106**, 1039-1061
- \_\_\_\_\_ and \_\_\_\_\_ (1992). "Advances in Prospect Theory: Cumulative Representation of Uncertainty," *Journal of Risk and Uncertainty* **5**, 297-323
- Tversky, Amos; Paul Slovic and Daniel Kahneman (1990). "The Causes of Preference Reversals," *American Economic Review* **80**, 204-217
- Tversky, Amos and Richard H. Thaler (1990). "Anomalies: Preference Reversals," *Journal of Economic Perspective* **4**(2), 201-211
- Tversky, Amos and Peter P. Wakker (1995). "Risk Attitudes and Decision Weights," *Econometrica* **63**, 1255-1280.
- von Neumann, John and Oskar Mogenstern (1944). *Theory of Games and Economic Behaviour*, Princeton University Press
- Wakker, Peter P. and Amos Tversky (1993). "An Axiomatization of Cumulative Prospect Theory," *Journal of Risk and Uncertainty* **7**, 147-176
- Wakker, Peter P. and Horst Zank (2002). "A Simple Preference Foundation of Cumulative Prospect Theory with Power Utility," *European Economic Review* **46**, 1253-1271
- Zank, Horst (2002). "Cumulative Prospect Theory for Parametric and Multiattribute Utilities," *Mathematics of Operations Research* **26**, 67-81