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**Modelling the Rand-Dollar Future Spot Rates:
The Kalman Filter Approach**

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Abstract

This paper examines the applicability of the Kalman filter technique to forecast the rand-dollar future spot rate. The failure of the "Unbiased forward rate hypothesis" in predicting the future spot rate conveys the very important information that participants in the rand-dollar forward market are risk averse and/or make use of adaptive expectation while forecasting the future spot rate for arbitrage or speculative motives. This paper also shows that policy change by the South African Reserve Bank is the cause for time-varying risk premium and time-varying coefficients in the model that predict the rand-dollar future spot rate. The better performance of the Kalman filter over the random walk and the time-invariant method (OLS) in out-of-sample forecasts confirms that a recursive technique with time-varying coefficients is relevant for forecasting the rand-dollar future spot rate.

1. Introduction

Forward exchange rates have traditionally been used as proxies for expected future spot rates, as most of the findings in the 1960s have supported the 'Unbiased Forward Rate Hypothesis' (UFRH). However, a large body of statistical work published since the introduction of flexible exchange rates in the early 1970s has established that forward rates for major currencies are not optimal predictors of future spot rates. For example, Hansen and Hodrick (1980 and 1983), and Agmon and Ahmihud (1981) found evidence of a risk premium in major forward foreign exchange markets, making the forward rate a biased predictor of the future spot rate.

Froot and Frankel (1989), though, concluded that variations in forward discounts indicate expected changes in exchange values rather than a risk premium and that the forward discount bias results primarily from irrationality by market participants in the foreign exchange market.

It is important to note that the unbiased forward rate hypothesis corroborates the efficient market hypothesis and, in its simplest form, the efficient market hypothesis can be reduced to a joint hypothesis that foreign exchange market participants are:

- a. Endowed with rational expectations
- b. Risk neutral

Therefore, from the joint hypothesis of the efficient market, a rejection of the UFRH may be either due to risk aversion from market participants or to a departure from the pure rational hypothesis, or to both of these reasons. The use of the time-invariant method, herein referred to as Ordinary Least Square (OLS), to explain the future spot rate from the forward rate becomes a source of model mis-specification if one considers the presence of risk premium from the risk aversion hypothesis and/or the absence of rational expectations from exchange market participants.

To account for risk aversion and adaptive expectations (in the absence of rational expectations), this study makes use of the linear state space model and the Kalman filter technique to forecast the rand-dollar future spot rate. The forecasting accuracy of the Kalman filter technique is compared with the OLS and Random walk method using different specification tests. The root mean square error (RMSE) and mean absolute error (MABE) are used to compare the accuracy of the three methods.

The Kalman filter technique has been used in a number of studies testing the Unbiased Forward Rate Hypothesis: for example, Albuquerque (1998) estimated the forward premium coefficient by accounting for the time-varying effects and conditional heteroscedasticity of the error using the Kalman filter. This study applied the Kalman filter technique for forecasting the future spot rate rand-dollar using the 6-month forward rate as a regressor. The study also demonstrated that policy change by the South African Reserve Bank, especially policy that impacts on the foreign exchange market, is the cause of time-variation premium as arbitrageurs and speculators continually revise their expectations given a certain policy by the central bank (Kohlhagen 1978).

The rest of the paper is organized as follows. The next section provides a theoretical background for the unbiasedness hypothesis and the shortcomings of the use of OLS method. Section III briefly describes the Kalman filter specification used in this study. The empirical results are reported and analyzed in section IV. The last section presents the conclusion.

2. Foreign Exchange Market Efficiency and the Unbiased Forward Rate Hypothesis

In an efficient speculative market prices should fully reflect information available to market participants and it should be impossible for a trader to earn excess return to speculation (Taylor & Sarno, 2002:5). Therefore, any arbitrage opportunity that presents in the foreign exchange market will quickly cancel out with the change in the conditions of supply and demand.

The forward rates, as with all derivatives products, are priced under the hypothesis of non-arbitrage opportunity. Assume $F_t^{(k)}$ is the k period's forward rate (the rate agreed now for an exchange of currency k periods ahead), S_t is the spot exchange rate (domestic price of foreign currency). If r and r_f represent the domestic and foreign interest rate respectively prevailing at time t, The fair price of the forward contract expressed in term of continuous compounding and under risk-neutrality hypothesis will then be:

$$F_t^{(k)} = S_t e^{(r-r_f)t} \quad (1). \text{ Under these conditions, any risk-free arbitrage opportunity cancels out.}$$

Expressed in terms of a natural logarithm, the expression becomes:

$$f_t^{(k)} - s_t = r - r_f \quad (2)$$

Where $f_t^{(k)} = \log_e F_t^{(k)}$ and $s_t = \log_e S_t$ and use has been made of the conventional expression $\log_e(1+x) \approx x$ for small x. So here $x = r - r_f$.

Expression (2) is known as the covered interest rate parity (CIP), and the reason why CIP should hold is that market deviation from expression (2) will result in arbitrage opportunity which will force the equality to hold.

If the risk-neutral efficient market hypothesis holds, then the expected foreign exchange gain from holding one currency rather than another, the expected exchange change, must be just offset by the opportunity cost of holding funds in this currency rather than the other, the interest rate differential. This condition is known as uncovered interest rate parity (UIP), expressed as:

$$\Delta_k S_{t+k}^e = r - r_f \quad (3)$$

Where $\Delta_k S_{t+k} = S_{t+k} - S_t$. expression (3) refers to the market expectation of the change in the spot price between time t and t+k.

Putting together expression (2) and (3), the covered and uncovered interest parity, one would then arrive at the conclusion that:

$f_t = E(s_{t+k}/I_t)$ (4), which implies that the forward rate at time t should be equal to the market expectation of the future rate, given information at time t .

Therefore expression (4) provides a basis for the testing of an unbiased forward rate hypothesis by estimating the following equation:

$$s_{t+k} = a + b f_t + h_{t+k} \quad (5)$$

h_{t+k} is the rational expectation forecast error with $E[h_{t+k}/I_t]=0$. To test the hypothesis that the forward rate is an unbiased predictor of the spot rate, the restriction $b=1$ is tested. A strong form of an unbiased market efficiency hypothesis and no risk premium implies testing $a=0$ (a constant risk premium equals zero) and $b=1$ and the errors are serially uncorrelated and homoscedastic.

Many estimations testing the unbiased forward rate hypothesis using the OLS method have rejected the hypothesis and many reasons have been advanced. For example Fama(1984) offers an explanation as to why the OLS coefficient $b < 0$. He argues that the rational expectation risk premium on foreign exchange rates must be extremely variable. McCallum (1994), however, provides different explanation. He explains the failure of the forward rate unbiasedness hypothesis by a neglect to take into account the fact that monetary authorities pursue interest rate smoothing and avoid exchange rate changes. Therefore there is a missing equation in an expression such as (5).

We are of the opinion that the activities of the central bank in the foreign exchange market would have an influence on the expectation of the change in spot rate by foreign exchange market participants. Therefore, market participants will cease to be risk neutral and will require a premium because of the wedge between interest rate differential and the expectation of the change in spot rate. A Policy shift from the central banks will then be the cause for time-varying risk premium. The OLS method, which estimates constant parameters, will therefore not be able to accommodate a time-varying coefficient explained by the variation of risk premium.

Moreover, change in policy from the monetary authority will cause foreign exchange market participants to continually learn about their environment and learning about the environment can generate forecast errors displaying serial correlation (Taylor *et al.*, 2003:28). This argument supports the view that agents in the foreign exchange market form their expectation of the future exchange rate, under the assumption of adaptive expectation rather than rational expectation. For example, they will revise their expectation of future spot rate upward if they had underpredicted in the past.

Given the argument of time-varying coefficients and adaptive expectations, the technique that will account for these two facts in modelling a given series will be the Kalman filter.

3. Kalman Filter approach

The Kalman filter is a recursive procedure for computing the optimal estimator of the state vector at time t , based on information available at time t (Harvey, 1989:104).

The Kalman filter provides a linear estimation method for equations represented in a state space form.

An estimation problem can be put into state-space form by defining the state vector represented by a certain parameter. The equation representing the state vector is known as the transition equation. The state vector is not observed directly; instead the state of the system is conveyed by an observed variable called signal equation, which is subject to contamination by disturbance or measurement error. The equation representing an observable variable is known as a measurement equation.

An example of the state-space model underlying the Kalman filter can be represented as follows:

$y_t = H_t x_t + h_t$ (6) representing the observation, signal or measurement equation

$x_t = \Phi_t x_{t-1} + u_t$ (7) representing the transition or state equation

y_t is therefore the observation on the system and x_t is the state vector. The random variables h_t and u_t represent the measurement and state disturbance or noise respectively. They are assumed to be independent of each other, white and with normal probability distribution, meaning that:

$$p(\mathbf{h}) \sim N(0, Q)$$

$$p(\mathbf{u}) \sim N(0, R)$$

$p(\mathbf{h})$ and $p(\mathbf{u})$ stand for probability distribution of the errors \mathbf{h} and \mathbf{u} respectively and Q is the covariance of the measurement while R is the covariance of the state noise.

Because of its recursive character, the estimation of the equations of the Kalman filter requires the determination of the initial estimate of the state vector x_0 at time $t=0$ and of its variance matrix. It is assumed that H_t , Φ_t , Q and R are known for all $t=1, \dots, n$, the same as the initial estimate for the state vector and its variance matrix. With a set of information at time t given as $I_t = \{y_1, \dots, y_t\}$ and given the initial estimate x_0 for the state vector x_0 , The Kalman filter equation determines the state vector estimates $x_{t|t-1} = E(x_t / I_{t-1})$ and $x_t = E(x_t / I_t)$ and their associated variance matrices.

A summary of the general Kalman filter equations of estimation and prediction can be presented as follows:

$$x_{t/t-1} = \Phi_t x_{t-1} \quad (10) \text{ State prediction}$$

$$P_{t/t-1} = \Phi_t P_{t-1} \Phi_t' + R_t \quad (11) \text{ Prediction dispersion}$$

$$e_t = y_t - H_t x_{t/t-1} \quad (12) \text{ Prediction error}$$

$$F_t = H_t P_{t/t-1} H_t' + Q_t \quad (13) \text{ Error dispersion}$$

$$K_t = P_{t/t-1} H_t' F_t^{-1} \quad (14) \text{ Kalman gain}$$

$$x_t = x_{t/t-1} + K_t e_t \quad (15) \text{ State estimate}$$

$$P_t = (I - K_t H_t) P_{t/t-1} \quad (16) \text{ Estimate dispersion}$$

The expression for the Kalman gain is given by: $K_t = P_t H_t' Q_t^{-1}$ (Pollock)

The elaboration of the recursive least square model which is required in order to achieve the generality of the Kalman filter, as explained above, may be described as follows:

$$y_t = \beta x_t + e \quad (17) \text{ for the measurement equation}$$

$$\beta_t = \Phi_t \beta_{t-1} + u_t \quad (18) \text{ for the state equation.}$$

The equations of the recursive least square follow the same characteristics of estimation as the general Kalman filter presented above. These equations are relevant for the estimation of models characterised by time-varying coefficients. Pagan (1980) shows that the time-varying coefficient model can be written into state space form so that the likelihood function can be easily calculated by the Kalman filter algorithm.

4. Data Analysis and Empirical Results

The aim of this paper, as outlined earlier, is to compare the forecast ability of the the following three techniques or methods for the determination of the future rand-dollar spot rate :

- a. The time-invariant (OLS) model
- b. The Kalman filter model
- c. The Random walk model

The OLS and Kalman filter estimate the future spot rate based on the unbiased forward rate hypothesis where the forward rate is the regressor.

The weekly closing rate for the spot rate and 6-month forward rate are collected from the I –Net Bridge historical data. The data cover the period from 28 January 1996 to 28 December 2003, giving a total of 414 observations. In all, 362 observations between 28 January 1996 and 21 December 2002 are utilized for parameter estimation and 52 observations thereafter are used for forecasting. The predictive accuracy of the model is tested over the forecast period (out-of-sample forecasting). Nevertheless mention will be made of the accuracy of the three techniques in-sample.

Each method will be dealt with in a separate subsection before comparison is made at the end of the three subsections.

4.1 The OLS method

The application of the OLS method aims at determining the relationship between the forward rate and future spot rate under assumption that the parameters do not vary in the sample considered (the time invariant parameter model). The accuracy of such an assumption has therefore to be tested. But before anything, to avoid estimating a spurious regression, the test of unit root is conducted to establish the level of integration of the two series and subsequently to establish if the two series are cointegrated.

Table 1 Augmented Dickey-Fuller (ADF) unit root test on D(Spot)

ADF Test Statistic	-8.719298	1% Critical Value*	-3.4485
		5% Critical Value	-2.8689
		10% Critical Value	-2.5707
*MacKinnon critical values for rejection of hypothesis of a unit root.			
Augmented Dickey-Fuller Test Equation			
Dependent Variable: D(SPOT,2)			
Method: Least Squares			
Sample(adjusted): 3/10/1996 12/28/2003			
Included observations: 408 after adjusting endpoints			

Table 2 ADF unit root test on D(forw6: 6-month forward rate)

ADF Test Statistic	-6.213279	1% Critical Value*	-3.4486
		5% Critical Value	-2.8689
		10% Critical Value	-2.5707
*MacKinnon critical values for rejection of hypothesis of a unit root.			
Augmented Dickey-Fuller Test Equation			
Dependent Variable: D(FORW6,2)			
Method: Least Squares			
Sample (adjusted): 3/31/1996 12/28/2003			
Included observations: 405 after adjusting endpoints			

The Dickey-Fuller test, especially the ADF test indicates that the two series are integrated at order 1, meaning that they are stationary after the first difference. The Akaike Information Criteria (AIC) established the lag of spot at 4 and the lag of forward at 7 for the ADF test. The null hypothesis of a unit root is rejected at the 1% level of significance.

Table 3 Johansen Cointegration test

Sample (adjusted): 3/03/1996 12/28/2003				
Included observations: 409 after adjusting endpoints				
Trend assumption: Linear deterministic trend				
Series: FORW6 SPOT				
Lags interval (in first differences): 1 to 4				
Unrestricted Cointegration Rank Test				
Hypothesized		Trace	5 Percent	1 Percent
No. of CE(s)	Eigenvalue	Statistic	Critical Value	Critical Value
None *	0.033273	16.16698	15.41	20.04
At most 1	0.005673	2.326680	3.76	6.65
*(**) denotes rejection of the hypothesis at the 5%(1%) level				
Trace test indicates 1 cointegrating equation(s) at the 5% level				
Trace test indicates no cointegration at the 1% level				

The Johansen test of cointegration and the Dickey-Fuller test of stationarity of residual confirm that the two series are cointegrated at the 5% level. Therefore we can proceed with the estimation of the relationship between the two variables.

The OLS estimation of the relationship between the future spot rate and the forward rate is represented in Table 4.

Table 4 Statistics of the OLS method

Dependent Variable: SPOT				
Method: Least Squares				
Sample(adjusted): 7/14/1996 12/22/2002				
Included observations: 337 after adjusting endpoints				
White Heteroskedasticity-Consistent Standard Errors & Covariance				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.049399	0.019043	2.594137	0.0099
FORW6(-24)	0.951680	0.025051	37.98971	0.0000
R-squared	0.867762	Mean dependent var		0.817547
Adjusted R-squared	0.867368	S.D. dependent var		0.126614
S.E. of regression	0.046111	Akaike info criterion		-3.309604
Sum squared resid	0.712292	Schwarz criterion		-3.286933
Log likelihood	559.6683	F-statistic		2198.318

This OLS estimation stems from the following equation:

$S_{t+k} = a + \beta f_t + \mu_{t+k}$, where $k=24$ for pairing the future spot rates with the appropriate forward rates.

The Breusch-Godfrey serial correlation LM test confirms that the residuals from the OLS regression are serially correlated. Also, the White's Heteroskedasticity test confirms the presence of heteroskedasticity in the residuals. The standard errors in the regression presented in Table 4 are corrected using the Newey-West Heteroskedasticity and Autocorrelation Consistent Variance (HAC).

The coefficients of the regression in Table 4 show that the null hypothesis of $a=0$ and $\beta=1$ are rejected. This shows support of the rejection of the Unbiased Forward Rate Hypothesis. Nevertheless, because our focus is on forecasting accuracy, emphasis will be put more on the statistical significance of the coefficients estimated.

The rejection of the UFRH confirms that participants in the rand-dollar foreign exchange market are not risk neutral and/or do not use rational expectations in their forecast of the future spot rate. As stated above, this is in part due to the policy change by the South African Reserve Bank. Activity and policy change by the SARB and their effect upon the movement of exchange rates has had different consequences on the behaviour of speculator and arbitrageur in the rand-dollar market. The reign of Christ Stal as a governor of reserve bank was characterised by a sort of dirty floating rate and the reign of Tito Mboweni shows that free-floating exchange is the core policy action in the foreign exchange market. Such a regime switching impacted on the expectation of the exchange rate formulated by market participants.

To test if there is a structural break in the forward premium between the reign of the two governors, use is made of Chow's breakpoint. The reference date used to test for the breakpoint in our sample is the 29 August 2000, the date the governor Tito Mboweni addressed shareholders at the general meeting of the SARB confirming the endorsement of inflation-targeting as a final policy target by the SARB and commitment to the principles of the free-floating system.

Table 5 Chow's breakpoint test

Chow Breakpoint Test: 9/03/2000			
F-statistic	38.70054	Probability	0.000000
Log likelihood ratio	71.27469	Probability	0.000000

The rejection of the null hypothesis of no structural change confirms that there is a structural change in the foreign exchange market of rand-dollar between the periods before and after August 2000. This supports the view that the time-variant coefficient has to do with policy switching by the SARB.

IV. 2 The Kalman Filter Estimation

The presence of breakpoints, as confirmed by the Chow breakpoint test, shows that coefficients estimated from equation (5) are not stable within the study sample. This supports the use of the time-varying coefficient technique in modelling the relationship between the future spot rate and forward rate. The Kalman filter will fit as the type of technique that accommodates time-varying coefficients and adaptive expectations.

To account for the time-varying risk premium and adaptive expectation, use is made of state space model with the application of the Kalman filter to estimate the future spot rate rand-dollar. The model specification is as follows:

$$S_{t+k} = a_t + \beta_t f_t + \mu_{t+k} \quad (19)$$

$$a_t = a_{t-1} + h_{1t} \quad (20)$$

$$\beta_t = \beta_{t-1} + h_{2t} \quad (21)$$

Equation (19) represents the measurement equation and equations (20) and (21) together describe the transition equations. These three equations represent the recursive system for modelling the future spot rate with the aid of the Kalman filter.

The E-views estimation of the state space model using Kalman filter estimation is presented in table 6.

Table 6 Statistics of the Kalman filter model

Sspace: SS02				
Method: Kalman filter				
Included observations: 361				
Valid observations: 337				
User prior mean: VECTOR01				
User prior variance: SYM01				
	Final State	Root MSE	z-Statistic	Prob.
SV1 (a_t)	0.050424	0.030073	1.676724	0.0936
SV2 (β_t)	0.950424	0.030073	31.60402	0.0000
Log likelihood	-318.1477	Akaike info criterion		1.888117
Parameters	0	Schwarz criterion		1.888117

Table 6 provides the estimates of the final state, To obtain these statistics the static values or initial conditions of the state space variables are set as follows:

- For the prior mean we use the coefficient obtain from the OLS estimation
- For the variance we choose a large number, 10000, as suggested by Harvey (1990:121-122), for a good approximation.

The result shows time variation of the parameters, at 1% level for β and 10% level for α from the final estimate. The result is consistent with the existence of time-varying risk premium.

4.3 Random Walk Model

Because the study compares the forecast ability of the OLS, Kalman Filter and Random walk estimation of the future spot, this section briefly present the estimation obtained from the Random Walk estimation of the future spot rate.

Table 7 provides the estimation of the model of the form:

$$s_t = s_{t-1} + h_t \quad (22)$$

The estimation obtained from Table 1 confirms that the AR(1) representation of the spot rate series depict a random walk process.

Table 7 Statistics of the Random walk model

Dependent Variable: SPOT				
Method: Least Squares				
Date: 04/27/04 Time: 11:02				
Sample(adjusted): 2/04/1996 12/28/2003				
Included observations: 413 after adjusting endpoints				
White Heteroskedasticity-Consistent Standard Errors & Covariance				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
SPOT(-1)	1.000595	0.000528	1895.078	0.0000
R-squared	0.995773	Mean dependent var		0.814014
Adjusted R-squared	0.995773	S.D. dependent var		0.126512
S.E. of regression	0.008225	Akaike info criterion		-6.760778
Sum squared resid	0.027874	Schwarz criterion		-6.751036
Log likelihood	1397.101	Durbin-Watson stat		1.757306

Table 7 represents the statistics of the random walk model after correcting the standard error by using White's heteroskedasticity and autocorrelation consistent covariance to account for the heteroskedasticity in the variance of the model.

5. Comparison of the models

Which model forecasts the best future rand-dollar spot rate?

This section provides the answer to the above question by comparing the in-sample and out-of-sample performance of each model. The basis for comparison will be the root means square error (RMSE) and the mean absolute error (MABE).

5.1. In-Sample performance

The results for the comparison of the performance of each of the three models is provided in table 8.

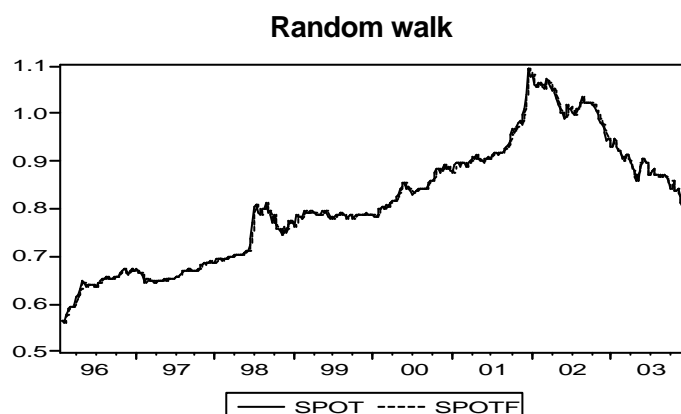
Table 8 In-Sample forecasting Accuracy

Model	RMSE	MABE
OLS	0.046097	0.034766
Random Walk	0.007515	0.004830
Kalman Filter	0.007931	0.024434

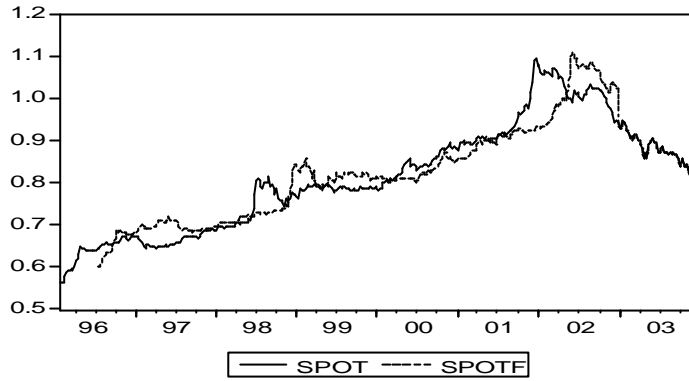
The in-sample has proven that the random walk model performed better than the the OLS and Kalman filter. The difference is not considerable between the random walk and Kalman filter as far as the root mean square root is concerned. Nevertheless, the Kalman filter performs better than the OLS method.

Figure 1 gives an idea on how the three methods have performed for the in-sample forecast.

Figure 1 In sample forecast

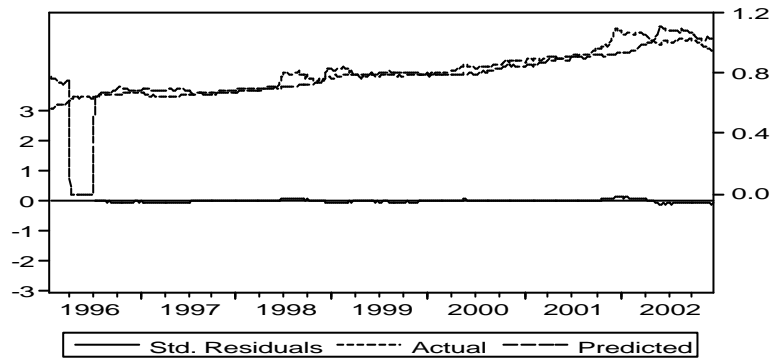


OLS



Kalman filter

One-step-ahead SPOT



5.2 out-of-sample performance

Table 9 provides the results for the out-of-sample performance.

Table 9 Out-of-sample statistics

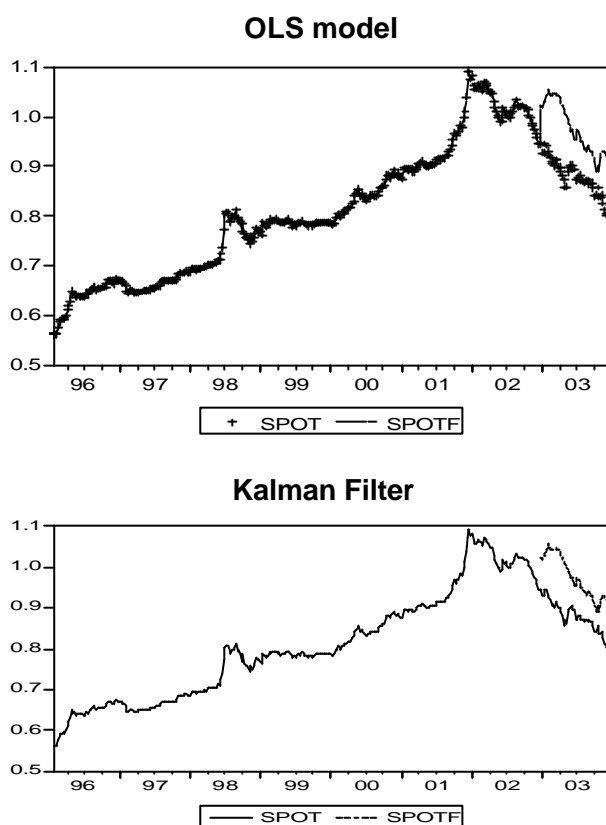
Model	RMSE	MABE
OLS	0.101117	0.096386
Random Walk	0.095344	0.084211
Kalman Filter	0.0100	0.0019

The out-of-sample shows that the Kalman filter model estimates the future spot rate better than the random walk and the OLS method. The OLS method remains the poorest method for the two types of forecasting. The decomposition of the mean squared forecast error for the OLS and Random Walk model reveals to us that there is a high bias proportion, telling

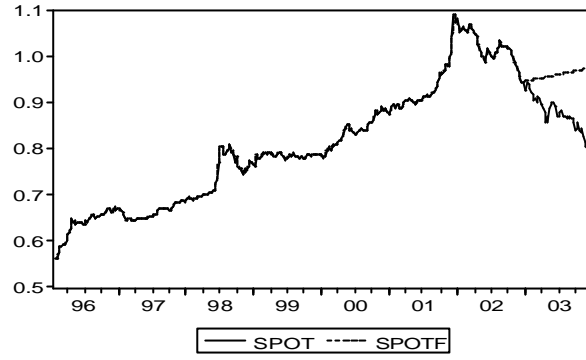
us how far the mean of the forecast is from the mean of the actual series. Close to 91% of the forecast error of the OLS is due to bias proportion and the number is 78% for the Random walk model. This supports the view that the expected value of the future rand-dollar spot rate is accurately determined if account is made of time-varying coefficients.

Figure 2 presents the out-of-sample of the three models.

Figure 8 Out-of- sample forecast



Random Walk



Note: Spot for actual and spot for predicted values.

Conclusion

This paper has attempted to model the relationship between the future spot rate and the forward rate for the rand-dollar exchange rate. With the failure of the unbiased forward rate hypothesis, traditionally seen as the best way to predict the future spot rate from the forward rate, this paper found that change of policy by the South African Reserve Bank is not only the recipe for risk premium by the foreign exchange market participants, but it is also seen as the reason for the variation of such premium. Market participants continually correct their expectation with a given policy by the SARB. These facts provide us with very important information: that for modeling the rand-dollar future spot rate account has to be taken of the time-varying coefficient instead of a constant parameter as used by the OLS method. The state space model with the application of The Kalman filter has been suggested as an alternative method to forecast the future spot rate.

Comparing the Kalman filter technique with the random walk and the constant parameter (OLS) method, we showed that the Kalman filter has performed better than the random walk and the OLS method out-of sample. The criterion for comparing the accuracy of the three methods was the root mean square error (RMSE) and the mean absolute error (MABE). The Kalman filter estimation of the relationship between the future spot rate and the forward rate shows that risk- premium is time-varying in the rand-dollar market.

References

- Agmon, T. and Y. Ahmihud. "The forward Exchange Rate and the prediction of the future spot rate," *Journal of banking and Finance*, Vol. 5 (September, 1981), 425-37.
- Albuquerque, R. (1998). "The Forward Premium Puzzle in as model of imperfect information: Theory and Evidence," Financial Research and policy working paper N0. Fr 00-09, The Bradley policy research center, University of Rochester.
- Bhakshi, G.S. and A. Naka. (1997). "Unbiasedness of the forward exchange rates" *The financial review*, Vol. 32(1), 145-162.
- Breusch, T. and A. Pagan. (1979). "A simple test of Heteroscedasticity and Random coefficient variation", *Econometrica*, Vol. 47, 327-336.
- Engel, C. (1996). "The Forward discount anomaly and the risk premium: A survey of recent evidence", *Journal of Empirical Finance*, Vol.3, 123-192.
- Felmingham, B.S. and P. Mansfield. "Rationality and the risk premium on the Australian dollar", *International Economic Journal*, Vol.11, Autumn 1997.
- Froot, K.A. and J.A. Frankel. "Forward Discount Bias: Is it an exchange risk premium?" *Quarterly journal of economics*, Vol.54 (February, 1989), 139-161.
- Hai, W., N.C. Mark, and Y. Wu. "Understanding spot and forward exchange rate regression", *Journal of applied econometrics*, Vol. 12, 715-734 (1997).
- Hansen, L.P. and R.J. Hodrick (1983), Risk averse speculation in the forward foreign exchange market : An econometric analysis of linear models", *Exchange rate and international macroeconomics*, University of Chicago Press for the National Bureau of Economic Research, Chicago.
- Harvey, A., E. Ruiz and N. Shepard. (1994). "Multivariate Stochastic Variance Model," *Review of Economic Studies*, Vol.61, 247-264.
- Harvey, A.C., (1990). *Forecasting, Structural Time series Models and the Kalman Filter*. Cambridge: Cambridge University Press.
- Hull, J.C. (2000). *Options, Futures, and other derivatives*, Fifth edition. London: Prentice Hall.
- Khal, D.R. and Ledolter, J. (1983). "A recursive Kalman Filter forecasting Approach" *Management Science*, Vol.29, 1325-1333
- Maybeck, P.S.,(1979). *Stochastic model, estimation, and control*. Volume I. London: Academic Press
- Pagan, A., (1980). "Some identification and estimation results for regression models with stochastically varying coefficient," *Journal of Econometrics*, Vol.13, 341-363.
- Phillips, P., (1986). "Understanding Spurious Regressions", *Journal of Econometrics*. Vol.33, 311-340
- Pollock, D.S.G., (2000). Notes on Recursive estimation and the Kalman Filter. www.google.com
- Swanson, P.E., (1998). "Spot and forward rate as predictor of future spot rates: Trend in exchange market value and the contribution of new information", *Journal of Economic and Finance*, 22, summer/fall 1998, 129-138.
- Taylor, M.P. and L. Sarno., (2002). *The Economics of exchange rates*. London: University Press.