Applied Time Series Econometrics

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Cointegration in Multivariate Systems

- Estimation by means of single equation techniques is becoming increasingly rare.
- Reason:
  - inability of single equation techniques to determine the number of cointegrating relationships present in the data,
  - the serious consequences of not taking into account the presence of >1 cointegrating relationship,
  - a means of establishing the number of cointegrating relationships in the data, and estimating in accordance is readily available.

- The Johansen technique (VECM) - has become the standard means of estimation in time series contexts. Proceeds by a number of distinct steps in estimation, including:

1. As before, testing the order of integration of each variable entering the multivariate model.
2. Selecting the correct lag length for the VAR model, so as to ensure that the $VECM$ has Gaussian errors.
3. Determining whether the system needs to be conditioned on any $I(0)$ variables, including dummy variables.
4. Testing for the reduced rank of the system. This includes the question of whether the system is $I(1)$ or $I(2)$.

5. Establishing whether the data contains trends, since this determines whether the system is to be estimated with deterministic variables (constant and trend) or not.
6. Testing for weak exogeneity.
7. Testing for linear hypotheses on cointegrating relationships.
8. Testing for unique cointegrating vectors.
9. Imposing joint tests of restrictions on the α loading matrix, and the β cointegrating vector.

- References:
  - Johansen (1991).
  - Johansen and Juselius (1990,1992).
  - Wickens (1996).

1. The Johansen Approach: an intuitive introduction

- Sims (1980): noted that in macroeconomic systems many variables are likely to be interdependent - rendering exogeneity of variables rare.
- Sims' suggested solution: have recourse to $Vector$ $Autoregressive$ estimation, referred to as the estimation of $VAR's$.

- To illustrate: a simple two variable VAR, with lag length 2 would be:

$$
\begin{aligned}
Y_t &= \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + \alpha_3 X_{t-1} + \alpha_4 X_{t-2} + u_{1t} \\
X_t &= \alpha_5 Y_{t-1} + \alpha_6 Y_{t-2} + \alpha_7 X_{t-1} + \alpha_8 X_{t-2} + u_{2t} \\
u_{it} &\sim IN\left(0, \sigma^2\right)
\end{aligned}
$$

where the lag length is determined by the need to render the error terms of the VAR such as to be $\sim IN\left(0, \sigma^2\right)$.

- A number of points are worth noting in connection with $VAR's$:
  - The VAR system does not impose strong *a priori* restrictions on:
    - any particular structural relation
    - the exogeneity of variables
  - The VAR system is effectively in *reduced form*, with only lagged values of variables appearing, so that all explanatory variables are weakly exogenous.

- We can generalize the specification of $VAR's$ as:

$$\underset{(n\times 1)}{Z_t} = \underset{(n\times n)}{A_1}\underset{(n\times 1)}{Z_{t-1}} + ... + \underset{(n\times n)}{A_k}\underset{(n\times 1)}{Z_{t-k}} + \underset{(n\times 1)}{u_t}$$

in vector notation, where as before $u_{it} \sim IN\left(0, \sigma^2\right)$.

- The generalized VAR can now be transformed into a set of difference equations:

$$\begin{aligned}\Delta Z_t &= \sum_{i=1}^{k-1} \Gamma_i \Delta Z_{t-i} + \Pi Z_{t-k} + u_t \\ &= \sum_{i=1}^{k-1} \Gamma_i \Delta Z_{t-i} + \alpha\beta' Z_{t-k} + u_t\end{aligned}$$

- The VAR expressed in the form of a set of difference equations provides us with two important bits of information:
  - First, it specifies what we might term the "seed" values from which the entire system evolves, given by the *level* of all variables in the initial period $t-k$, which we have denoted by Z_{t-k}.
  - Second, it specifies all subsequent *changes* in the variables from the initial "seed" values given by Z_{t-k}, through the difference terms which appear on the RHS of the set of difference equations, viz. the $\Delta Z_{t-1}, ..., \Delta Z_{t-k+1}$.

- The J-A in effect proceeds by an estimation of the set of difference equations specified above.

  $\Longrightarrow$ represents an important advance over the single equation estimation techniques such as the E-G:

  - Generally single equation techniques ignored the presence and effect of short run-dynamics on the relationship to be estimated.

- The J-A, by having recourse to estimation of the difference equation above avoids this limitation:
  * recognises both the importance of relationships between the variables included in the system in *levels*, allowing us to obtain an intimation of the equilibrium relationship between the variables.
  * and the evolution of the system of variables over subsequent time periods, thus allowing us to capture the characteristics of the short run dynamics of the system.

- The standard J-A applies only to systems of variables which are at most $I(1)$. Recall:

$$\Delta Z_t = \sum_{i=1}^{k-1} \Gamma_i \Delta Z_{t-i} + \alpha\beta' Z_{t-k} + u_t$$

  - By assumption $u_t \sim IN\left(0, \sigma^2\right) \sim I(0)$.
  - If all the variables in the system of equations are $I(1)$, then it follows that all $\Delta Z_{t-i} \sim I(0)$.
  - Hence the only remaining term in the set of difference equations, $\Pi Z_{t-k} \sim I(0)$:
    * because cointegrated: the ΠZ_{t-k} term, in order to be $\sim I(0)$, must be such that it must contain within it *cointegrating relationships between the variables in levels*.
    * because: $\Pi = \varnothing$, trivial case.
    * because: $Z_{t-k} \sim I(0)$, trivial case.

2. The Johansen Method of Reduced Rank Regression

- We can now rewrite:

  $\Delta Z_t = \Gamma_1 \Delta Z_{t-1} + ... + \Gamma_{k-1} \Delta Z_{t-k+1} + \Pi Z_{t-k} + u_t$

  as:

  $\Delta Z_t - \alpha\beta' Z_{t-k} = \Gamma_1 \Delta Z_{t-1} + ... + \Gamma_{k-1} \Delta Z_{t-k+1} + u_t$

  allowing us to run two separate regressions, separating the short run dynamics from the equilibrium relationship:

$$\begin{aligned} \Delta Z_t &= M_1 \Delta Z_{t-1} + ... + M_{k-1} \Delta Z_{t-k+1} + R_{0t} \\ Z_{t-k} &= N_1 \Delta Z_{t-1} + ... + N_{k-1} \Delta Z_{t-k+1} + R_{kt} \end{aligned}$$

- Generating the two vectors R_{0t} and R_{kt} which may be used to form the residual product moment matrices:

$$S_{ij} = T^{1} \sum_{t=0}^{T} R_{it} R_{jt} \quad , \quad i, j = 0, k \qquad (1)$$

The ML estimate of β are the eigenvectors corresponding to the r largest eigenvalues from:

$$\mid \lambda S_{kk} - S_{k0} S_{00}^{-1} S_{0k} \mid = 0 \qquad (2)$$

- β simultaneously:
  - diagonalizes S_{kk} to a unit matrix
  - diagonalizes $S_{k0}S_{00}^{-1}S_{0k}$ to Λ the matrix of eigen-values

  $\Longrightarrow$ two diagonalizations impose r^2 restrictions

- $\hat{\beta}$ provides the n eigenvalues $\hat{\lambda}_1 > \hat{\lambda}_2 > ... > \hat{\lambda}_n$ and corresponding eigenvectors $\hat{V} = \left(\hat{v}_1, ..., \hat{v}_n\right)$.
- Since the eigenvalues are the *largest* squared canonical correlations between the "levels" residuals R_{kt} and the "difference" residuals R_{0t}, we obtain estimates of all the distinct *combinations of the* $I(1)$ *levels of* Z_t which produce high correlations with the *stationary* $I(0)$ *first difference terms* ΔZ_t.
- Such combinations must constitute the cointegrating vectors since they must themselves be $I(0)$ in order to achieve a high correlation with the stationary difference terms.

- The magnitude of $\hat{\lambda}_i$ is a measure of the strength of correlation between the cointegrating relations $\hat{v}_i' Z_t$ for $i = 1...r$ (or in terms of standard notation, the $\beta_i' Z_t$) and the stationary difference component of the model.

- The remaining $n - r$ combinations obtained as solutions to the preceding equation, (viz. the $\hat{v}_i' Z_t$ for $i = r + 1...n$) represent the nonstationary combinations of the Z_t which therefore prove to be uncorrelated with the stationary difference terms.

- Hence, $\hat{\lambda}_i = 0$ for $i = r + 1...n$.

- Johansen (1992b) points out that the test for the number of cointegrating vectors present in a set of data, denoted by r, amounts to a test that $\hat{\lambda}_{r+1} = \hat{\lambda}_{r+2} = ... = \hat{\lambda}_n = 0$, while $\hat{\lambda}_1 > \hat{\lambda}_2 > ... > \hat{\lambda}_r > 0$. Since

- Johansen (1992) demonstrates that $\hat{\lambda}_i = \hat{\alpha}_i' S_{00}^{-1} \alpha_i$, we can test for $\hat{\lambda}_{r+1} = \hat{\lambda}_{r+2} = ... = \hat{\lambda}_n = 0$ by testing whether the corresponding eigenvalues are insignificantly small (i.e. statistically equivalent to 0).

- Thus the test for the number of cointegrating vectors present in the data amounts to a test for the $n - r$ rows of the α matrix which are zero.

  $\Longrightarrow$ the test for the number of r cointegrating vectors present in β is effectively a consideration of the number of r linearly independent columns in Π, or the rank r of Π.

- Thus to recap:
  - Given $u_t \sim I(0)$,
  - For the relationship between variables to be meaningful requires that Πz_{t-k} contain stationary long-run relationships.
  - This occurs where $\Pi = \alpha\beta'$ has reduced rank, such that there exist $r \leq (n-1)$ cointegration vectors present in β.
  - This requires establishing the number r of linearly independent columns of Π, or the $(n-r)$ rows of α which are insignificantly different from 0.
  - Johansen's ML approach involves a reduced-rank regression, providing n eigenvalues $\widehat{\lambda}_1 > \widehat{\lambda}_2 > ... > \widehat{\lambda}_n$ with their corresponding eigenvectors $\widehat{V} = (\widehat{v}_1, ..., \widehat{v}_n)$.

- The r elements of $\widehat{V}$ that produce the $\widehat{v}_i' z_{t-k}$ $(i = 1, ..., r)$ linear combinations of the $I(1)$ levels of z_{t-k} which show high correlations with the stationary $\Delta z_t \sim I(0)$ are the cointegration vectors since in order to generate high correlations they must themselves be $\sim I(0)$.
- Since each eigenvector $\widehat{v}_i$ has a corresponding eigenvalue, the magnitude of $\widehat{\lambda}_i$ is a measure of the strength with which the cointegrating relations, $\widehat{v}_i' z_{t-k}$ denoted $\widehat{\beta}_i' z_{t-k}$, are correlated with the stationary part of the model.
- In effect, we expect the first r cases of $\widehat{\lambda}_i$ to be "large" (stat. sig. different from 0), and the last $(n - r)$ cases of $\widehat{\lambda}_i$ to be "small" (stat. insig. different from 0).

- The test of the null hypothesis that there are r cointegrating vectors present therefore can be stated as:

$$H_0 : \lambda_i = 0 \quad i = r + 1, ..., n \tag{3}$$

where the restriction may be imposed for different values of r.

- The log of the maximised likelihood function for the restricted model can then be compared to the log of the maximised likelihood function for the unrestricted model in order to construct a standard likelihood ratio test - subject to a nonstandard distribution.

- Two alternative formulations are possible:
  - The *trace* statistic can be computed as:

$$\lambda_{trace} = -2\log\left(\frac{restricted\ maximised\ likelihood}{unrestricted\ maximised\ likelihood}\right)$$
$$= -T\sum_{i=r+1}^{n}\log\left(1-\widehat{\lambda}_i\right) \quad r = 0, 1, 2, ..., n-1$$

  - The *maximal-eigenvalue* or $\lambda - \max$ statistic is given by:

$$\lambda_{\max} = -T\log\left(1-\widehat{\lambda}_{r+1}\right) \quad r = 0, 1, 2, ..., n-1$$

    testing the null of r cointegration vectors against the alternative of $r+1$.

- Asymptotic critical values appear in Osterwald-Lenum (1992):
  - the inclusion of dummy variables renders these critical values indicative rather than conclusive.
  - small samples renders the power and size properties of the test statistics problematic.

- A number of important points follow immediately from this discussion:

1. Impact of including $I(0)$ variables:
   (a) Every stationary $I(0)$ variable included in the Z_t vector would form a linearly independent relation on its own.
   (b) Where there is more than one $I(0)$ variable in Z_t, any linear combination of the $I(0)$ variables would again prove to be stationary.
   $\Longrightarrow$ Knowledge of the integration properties of the data is critical in being able to interpret the output from the J-A.

2. Where the model contains $I(2)$ variables, do not have linear combinations of variables which are themselves $I(0)$, with strong correlation with the $I(0)$ first difference terms.

   $\Longrightarrow$ data in first difference form

   $\Longrightarrow$ use the Johansen (1994) $I(2)$ technique.

3. In the presence of a small sample, Reimers (1992) suggests that the test statistics overreject the null.

   $\Longrightarrow$ adjustment: replacing T by $(T - nk)$ in *trace* and $\lambda - \max$, where T denotes sample size, n the number of variables, and k the lag length set for the estimation of VAR.

4. Monte Carlo Studies $\Longrightarrow$ trace statistic is more robust to both skewness and excess kurtosis in residuals than the maximal eigenvalue test.
5. At this stage we have determined only the *number* of unique cointegrating vectors that span the cointegration space (are present in the data).

   $\Longrightarrow$ Establishing the number of unique cointegrating vectors, does not establish that the estimated vectors themselves are unique: since $\alpha\beta' = \alpha\eta^{-1}\eta\beta' = \alpha^*\beta^*$ where η is any $r \times r$ nonsingular matrix.
6. Useful application of J-A,

   $\Longrightarrow$ must be able to establish *unique structural relationships* between variables, $\Longrightarrow$ do specific and economically meaningful relationships span the cointegration space?

2. The dynamic specification fo the model

- – We now proceed to the question of how to formulate the model to be estimated.
- Important questions in this regard:
  - Should the model contain deterministic components or not? (intercepts, trends, or seasonal dummies).
  - What is the appropriate lag length for the dynamic components of the model?

2.1 Inclusion of Deterministic Components

- Consider:

$$\Delta Z_t = \sum_{i=1}^{k+1} \Gamma_i \Delta Z_{t-i} + \alpha \begin{bmatrix} \beta \\ \mu_1 \\ \delta_1 \end{bmatrix} \begin{bmatrix} Z_{t-k} & 1 & t \end{bmatrix} + \alpha_\perp \mu_2 + \alpha_\perp \delta_2 t + u_t \tag{4}$$

Only where $\mu_1 = \mu_2 = \delta_1 = \delta_2 = 0$ will there be no deterministic elements in either the data or the cointegrating relations.

- Since this is unlikely to ever occur in practice, we need to consider the possible alternatives:
  - No linear trends are present in the data, $\Longrightarrow$ first-differences have zero mean,
    $\Longrightarrow \mu_2 = \delta_1 = \delta_2 = 0.$
    $\Longrightarrow$ an intercept will appear *only in the long run model*,
    $\Longrightarrow$ accounts for the units of measurement of the variables in Z_{t-k}.
  - Where there are trends in the levels of the data, $\Longrightarrow$ allow for drift in the non-stationary elements of the model (i.e. the short run model),
    $\Longrightarrow \delta_1 = \delta_2 = 0.$
    $\Longrightarrow$ intercept will be included only in the short run model, not in the long run model.

– Where there is some long run linear growth which cannot be accounted for by the model

$\Longrightarrow$ a linear trend in the long run model.

$\Longrightarrow \delta_2 = 0$.

$\Longrightarrow$ justification for the inclusion of time: specified model may have excluded an exogenous determinant of variables in the model, eg. technical progress.

– Where linear trends are included in the short run model,

$\Longrightarrow \delta_2 \neq 0$, $\Longrightarrow$ quadratic trends in Z_t.

$\Longrightarrow$ difficult to justify on *a priori* economic grounds,

$\Longrightarrow$ seldom considered as a viable alternative in practice.

- Which of these alternatives should be chosen? Johansen (1992c) suggests using the Pantula principle, which proceeds by:
  - estimating all three models: $\mu_2 = \delta_1 = \delta_2 = 0$, $\delta_1 = \delta_2 = 0$, and $\delta_2 = 0$;
  - presenting results from the most restrictive alternative (where $r = 0$ and $\mu_2 = \delta_1 = \delta_2 = 0$) to the least restrictive alternative (where $r = n - 1$ and $\delta_2 = 0$);

– we now compare the trace or maximal eigenvalue test statistic against their critical values, and proceed to ever lower levels of restrictiveness on deterministic components, until the null can no longer be rejected;

– at this point we stop and accept the corresponding number of cointegrating vectors and the associated deterministic components.

2.2 Appropriate lag length

- The criterion for choosing appropriate lag length is simple: the errors must be rendered Gaussian.
- Check the characteristics of the errors of the system of equations, equation by equation, to ensure that the errors are indeed well-behaved.

- Choice of lag length is also associated with the question of whether the short run dynamics of the system is affected by variables not relevant to the long run equilirium relationship that is to be estimated.
- Should they be omitted, they would become part of the error term of the model, and generate “systematic patterns” in the residuals of the estimation.
- Under such circumstances the appropriate response is *not* the extension of lag length (to mop up additional autocorrelation, say) but to revert to an appropriate specification of the dynamics of the model.

3. Imposing and Testing for Restrictions on Parameters

- The simple identification of the number of cointegrating vectors present in our data, and estimating the cointegrating vectors is not yet sufficient.
- Reason: the estimation of the cointegrating vectors present in the data suffices only to establish the cointegrating space - it does not yet establish that the cointegrating vectors that we have estimated are either unique, or whether the estimated vectors conform to the priors we derive from economic theory.

3.1 Weak Exogeneity

- Noted that the Π matrix decomposes into $\alpha\beta'$, where the β matrix contains information about the long run coefficients and hence the cointegrating vectors, and the α matrix contains information about the speed of adjustment to disequilibrium.
- Further, we noted that the finding that $r \leq (n-1)$ cointegrating vectors are present in the data implied that the last $(n-r)$ rows of α are zero.
- We turn now to a brief discussion of the non-zero elements of the α matrix, and the information we can glean from them for purposes of interpreting our results.

- Consider the very simple VECM given by:

$$\begin{bmatrix} \Delta y_{1t} \\ \Delta y_{2t} \\ \Delta y_{3t} \end{bmatrix} = \Gamma_1 \begin{bmatrix} \Delta y_{1t-1} \\ \Delta y_{2t-1} \\ \Delta y_{3t-1} \end{bmatrix} + \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{bmatrix} \begin{bmatrix} \beta_{11} & \beta_{21} & \beta_{31} \\ \beta_{12} & \beta_{22} & \beta_{32} \end{bmatrix} \begin{bmatrix} y_{1t-2} \\ y_{2t-2} \\ y_{3t-2} \end{bmatrix} + u_t$$

- Suppose that $r = 2$, such that there are two cointegrating vectors present in the data.

  $\Longrightarrow$ for instance $\alpha_{31} = \alpha_{32} = 0$ (though it could of course also be either of the other pairs, α_{11}, α_{12}, and α_{21}, α_{22})

- and as a consequence the equation in the VECM for Δy_{3t} would contain no information for the long run relationships present in the data.
- Note that this does *not* mean that the y_3 variable does not feature in the long run relationships, since it would still appear in the Δy_{1t} and Δy_{2t} equations.

- Strictly speaking, therefore, all that is required of y_{3t} is that it appear on the right-hand side of the VECM, and we could rewrite as:

$$\begin{bmatrix} \Delta y_{1t} \\ \Delta y_{2t} \end{bmatrix} = \Gamma_0 \Delta y_{3t} + \Gamma_1 \begin{bmatrix} \Delta y_{1t-1} \\ \Delta y_{2t-1} \\ \Delta y_{3t-1} \end{bmatrix} + \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \beta_{11} & \beta_{21} & \beta_{31} \\ \beta_{12} & \beta_{22} & \beta_{32} \end{bmatrix} \begin{bmatrix} y_{1t-2} \\ y_{2t-2} \\ y_{3t-2} \end{bmatrix}$$

- So what is the point of establishing the weak exogeneity properties of the system of equations we are estimating?:
  - Should all "problematic" data features (discussed under the section on dynamics and the specification of lag length in the VECM) be associated with the weakly exogenous variables, then by conditioning on the weakly exogenous variables we will be able to provide the rest of the system driving the Δy_{it} with better stochastic properties - i.e. the residuals of the short run VECM will be more devoid of problems.
  - In the presence of weak exogeneity, the VECM becomes more parsimonious (in the sense that it comes to contain fewer variables, and thus is easier to estimate).

- However, it should be noted that it is not desirable to start estimation with the weak exogeneity conditioning assumptions in place. It is preferable to start with a full specification, and to then test for the presence of weak exogeneity in the system.
- Moreover, we should note that the presence of the weak exogeneity restrictions on the loading matrix will alter (again) the asymptotic distributions of the rank test statistics from those that apply to the full model.
- As a consequence, it is simply easier, and the convention, to estimate the restricted model after the full model has been employed to determine which restrictions should be placed on both α and β.

- The test for weak exogeneity in the system as a whole requires a test of whether a specific row, say the $i'th$, contain only zeros:

$$H: \ \alpha_{ij} = 0 \quad , \quad j = 1, ..., r \tag{5}$$

- We can conduct the test by placing the zero restriction on the relevant row, and then proceeding to estimate the restricted model.
- Estimation of both the restricted and the unrestricted model, allows us to construct a likelihood ratio test.

- We thereby obtain $n-1$ eigenvalues from the unrestricted model, denoted $\widehat{\lambda}_i$, and $n-1$ eigenvalues from the restricted model, denoted $\widehat{\lambda}_i^*$. This allows us to construct the LR test statistic:

$$-2\log(Q) = T\sum_{i=1}^{r}\log\left[\frac{1-\widehat{\lambda}_i^*}{1-\widehat{\lambda}_i}\right] \sim \chi^2\left(r\times(n-m)\right)$$

where m denotes the number of variables in the system of equations, in order to test the null: that the zero restriction(s) on the α-rows are valid.

- Again, as for the trace and maximal eigenvalue test statistics, we note the possible need for imposing a small sample correction to this test, thus controlling for degrees of freedom. To do so, we use:

  $$-2\log(Q) = \left\{T - \frac{l}{n}\right\} \sum_{i=1}^{r} \log\left[\frac{1-\widehat{\lambda}_i^*}{1-\widehat{\lambda}_i}\right] \sim \chi^2\left(r \times (n-m)\right)$$

  where l denotes the number of parameters estimated in the reduced rank regression model.

3.2 Testing for Linear Restrictions on Cointegrating Vectors

- The procedure we employ is to impose restrictions on the cointegrating space that are motivated by economic theory:
  - such that some parameters in the long run economic relationships are expected to be such that $\beta_{ij} = 0$,
  - or that they satisfy homogeneity restrictions, for instance that $\beta_{3j} = \beta_{4j}$.
- Once the restrictions are imposed, we proceed to test whether the cointegrating vectors are identified.
- This will result in a series of LR statistics $\sim \chi^2(df)$.

3.3 Identification, and Testing for Unique Cointegrating Vectors

- Thus far we have not addressed the question of how to establish whether the cointegrating vectors within the unique cointegrating space contained within our data are themselves unique.
- In order to establish the uniqueness of the cointegrating vectors, we need to impose restrictions on the cointegrating vectors.
- We have just learnt to impose restrictions on the β matrix.

- As always in estimation contexts, our prior would be to anticipate the relevant restrictions to emerge from economic theory.
- The last step that now remains is to test whether the individual columns of β, and hence the distinct cointegrating vectors are identified.

- It is identification that establishes the uniqueness of the cointegration vectors. Johansen (1992d) defines identification as occurring when:

  it is not possible by taking linear combinations of for instance $\beta_2, ..., \beta_r$ to construct a vector and hence an equation which is restricted in the same way as β_1 and in this sense could be confused with the equation defined by β_1. Hence, $\beta_1$can be recognised among all linear combinations of $\beta_1, ..., \beta_r$ as the only one that is in sp(H_1), the space spanned by the columns in H_1 or the only one that satisfies the restrictions R_1.

- In effect the requirement for identification here is no more and no less than that we require in all simultaneous equation estimation.
- Whenever we have systems of equations, in order to "identify" what is forcing what, we need to have independent variation in at least some variables, such that we can observe which other variables react to the forcing variables, how and to what extent.

- Pesaran and Shin (1994) establish a set of formal conditions for identification to parallel the Johansen form. The procedure rests on the identification of long run terms and dynamics separately. In particular, they establish the need for $k = r^2$ restrictions a priori, where r denotes the number of cointegrating vectors. Moreover:
  - $k < r^2$ implies the model is unidentified
  - $k = r^2$ implies the model is exactly identified
  - $k > r^2$ implies the model is overidentified

- Greenside, Hall and Henry (1998) suggest an additional way forward in:

$$\Delta Z_t = \sum_{i=1}^{p-1} \Gamma_i \Delta Z_{t-i} + \Pi Z_{t-p} + v_t$$

  They point out that the provision of the over-identifying restrictions can come from one of four possible sources (see Greenside et al, 1998:3ff):

  - Restrictions on the cointegrating rank of Π, $r < N$
  - Restrictions on the dynamic path of adjustment (the Γ_i)
  - Restrictions on the cointegrating vectors, the β of $\Pi = \alpha\beta'$
  - Restrictions on the exogeneity or long run causality of the system, the α

- Thus we have the option of either:
  - imposing the number of cointegrating vectors on the data in the light of prior theory.
  - employing identifying restrictions on the β matrix in the manner outlined above.
  - specifying specific variables to be exogenous on the basis of theoretical priors.
  - or employing restrictions on the dynamics (the Γ matrixes) in order to obtain identification.

3.4 Impulse Responses

- One advantage of cointegration analysis is that the explicit consideration of dynamics that it offers allows us to examine the effects of shocks on variables of interest.
- We do so by means of impulse response functions, that are readily derived from VAR-structures.
- Recall from the preceding discussion that we can write our system of equations as

$$\Delta Z_t = \mu + \sum_{i=1}^{k-1} \Gamma_i \Delta Z_{t-i} + \Pi Z_{t-k} + u_t \qquad (6)$$

- The impulse response function can now be derived on the basis of an estimation of equation 6. We reparameterize such that:

$$
\begin{aligned}
Z_t &= \mu + \sum_{i=1}^{k} \Theta_i Z_{t-i} + u_t \qquad (7)\\
\Theta_1 &= \underset{(n \times n)}{I} + \widehat{\Gamma}_1 \\
\Theta_i &= \widehat{\Gamma}_i - \widehat{\Gamma}_{i-1} \ , \ \forall \ i = 2.....(k-1) \\
\Theta_k &= \widehat{\Gamma}_{k-1} + \Pi
\end{aligned}
$$

- The impluse response function can now be computed from the Θ_i coefficients as

$$\begin{aligned} \phi_i &= \sum_{j=1}^{k} \phi_{i-j}\Theta_j \ , \ \ i = 1, 2..... \qquad (8) \\ \Theta_j &= 0 \ for \ j > k \\ \phi_i &= 0 \ for \ i < 0 \end{aligned}$$

where the ϕ_i incorporate the estimated net impact of the stochastic disturbances that occur throughout the system of equations (and strictly speaking, the impact of the deterministic trend that is introduced by any deterministic components present in the cointegrating relationships), on each of the equations of the system.

- To be precise, for the system of equations (which is a little more general than that of equation 6, in that it incorporates trend and deterministic elements):

$$\Delta Z_t = \mu + at + \psi W_t + \sum_{i=1}^{k-1} \Gamma_i \Delta Z_{t-i} + \Pi Z_{t-k} + u_t \tag{9}$$

where W_t is a vector of exogenous/deterministic $I(0)$ variables,

- The solution to equation 9 is given by:

$$
\begin{aligned}
Z_t &= Z_0 + b_0 t + b_1 \left[\frac{t(t+1)}{2}\right] + C(1) S_t + C^*(L)(h_t - h_0) \\
h_t &= \psi W_t + u_t \\
S_t &= \sum_{i=1}^{t} u_i \ , \ t = 1, 2..... \\
b_0 &= C(1)\mu + C^*(1) a \\
b_1 &= C(1) a \\
C(L) &= C(1) + (1 - L) C^*(L) \\
C^*(L) &= \sum_{i=0}^{\infty} C_i^* L^i
\end{aligned}
$$

where L is the one-period lag operator.

- It is possible to show that:

$$\phi_i = C(1) + C_i^* \tag{10}$$

confirming the interpretation of the ϕ_i coefficient as the net impact of the random shocks that occur throughout the system of equations.

- It is worth noting at this point that:
  - In terms of the impact of a random shock on *individual variables*, $\phi_i \neq 0$. Since from equation 10 $\lim_{i \to \infty} \phi_i = C(1)$.
  - In terms of the impact of a random shock on legitimate *cointegrating vectors*, the impact has to be zero, else they would not represent equilibrium relationships. The information contained in the impluse response for the cointegrating vector, effectively contains information about the speed of, and nature of convergence to equilibrium.

– This is not true of the impact of a random shock on *constituent variables* within the cointegrating relationships.
– Generalized impulse response better than orthogonalized: since not senistive to order in which variables enter the system.

4. Example: Political Institutions & Economic Growth

- Fedderke (1999): possibility of simultaneity between rights and economic activity provides the postulate that:

$$Y = R(\bullet) Q(\bullet) \quad (11)$$

$$Q(\bullet) = K^{\alpha} L^{1-\alpha} \quad (12)$$

  assuming Cobb-Douglas technology under homogeneity of degree one. This left two possible cases to be distinguished:

- Where rights impact on output, and *rights are held to be determined by the technology of production* available, say:

$$R = q^{\gamma} = k^{\alpha\gamma} \tag{13}$$

so that per capita output is given by:

$$y = k^{\alpha(1+\gamma)} \tag{14}$$

and the growth rate of per capita output would now be:

$$\overset{\bullet}{y} = \alpha\left(1+\gamma\right)\overset{\bullet}{k} \tag{15}$$

- Here the rights-technology nexus exercises an influence on the growth rate of per capita output (in contrast to simple modernization theory), and the logical possibility of unbounded (endogenous) growth emerges where $\gamma > \frac{1}{\alpha} - 1$, though this remains a fairly unlikely occurrence given standard α-parameter values (the γ-parameter would have to be large, > 2 for most estimates of α).

- Where rights impact on output, and *rights are held to be determined by per capita output*, say:

$$R = y^{\gamma} \tag{16}$$

so that given equations 11 and 12:

$$y = k^{\frac{\alpha}{1-\gamma}} \tag{17}$$

and hence:

$$\overset{\bullet}{y} = \left(\frac{\alpha}{1-\gamma}\right)\overset{\bullet}{k} \tag{18}$$

- Again the rights-output nexus exercises an influence on the growth rate of per capita output, and the possibility of unbounded growth arises for $1 - \alpha < \gamma < 1$ (a much weaker requirement than $\gamma > \frac{1}{\alpha} - 1$).
- Interestingly, the case of $\gamma > 1$ also brings with it the possibility of low level institutional traps.

- Gives the VECM specification:

$$\Delta z_t = \sum_{i=1}^{k-1} \Gamma_i \Delta z_{t-i} + \Pi z_{t-k+1} + \mu + \delta_t \qquad (19)$$

The existence of r cointegrating relationships amounts to the hypothesis that:

$$H_1(r) : \Pi = \alpha\beta' \qquad (20)$$

where Π is $p \times p$, and α, β are $p \times r$ matrices of full rank. $H_1(r)$ is thus the hypothesis of reduced rank of Π. Where $r > 1$, issues of identification arise.

- In the cases identified in the brief theoretical discussion above we expect $r = 2$, and for the long run parameters:

$$\Pi z_{t-k+1} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{bmatrix} \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \end{bmatrix} \begin{bmatrix} y \\ k \\ r \end{bmatrix}_{t-k+1}$$

- Exact identification requires r^2 restrictions, for the expectation that $r = 2$ thus 4.
- On the basis of the discussion above we specify:

$$\Pi z_{t-k+1} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{bmatrix} \begin{bmatrix} 1 & -\beta_{12} & 0 \\ 0 & -\beta_{22} & 1 \end{bmatrix} \begin{bmatrix} y \\ k \\ r \end{bmatrix}_{t-k+1}$$

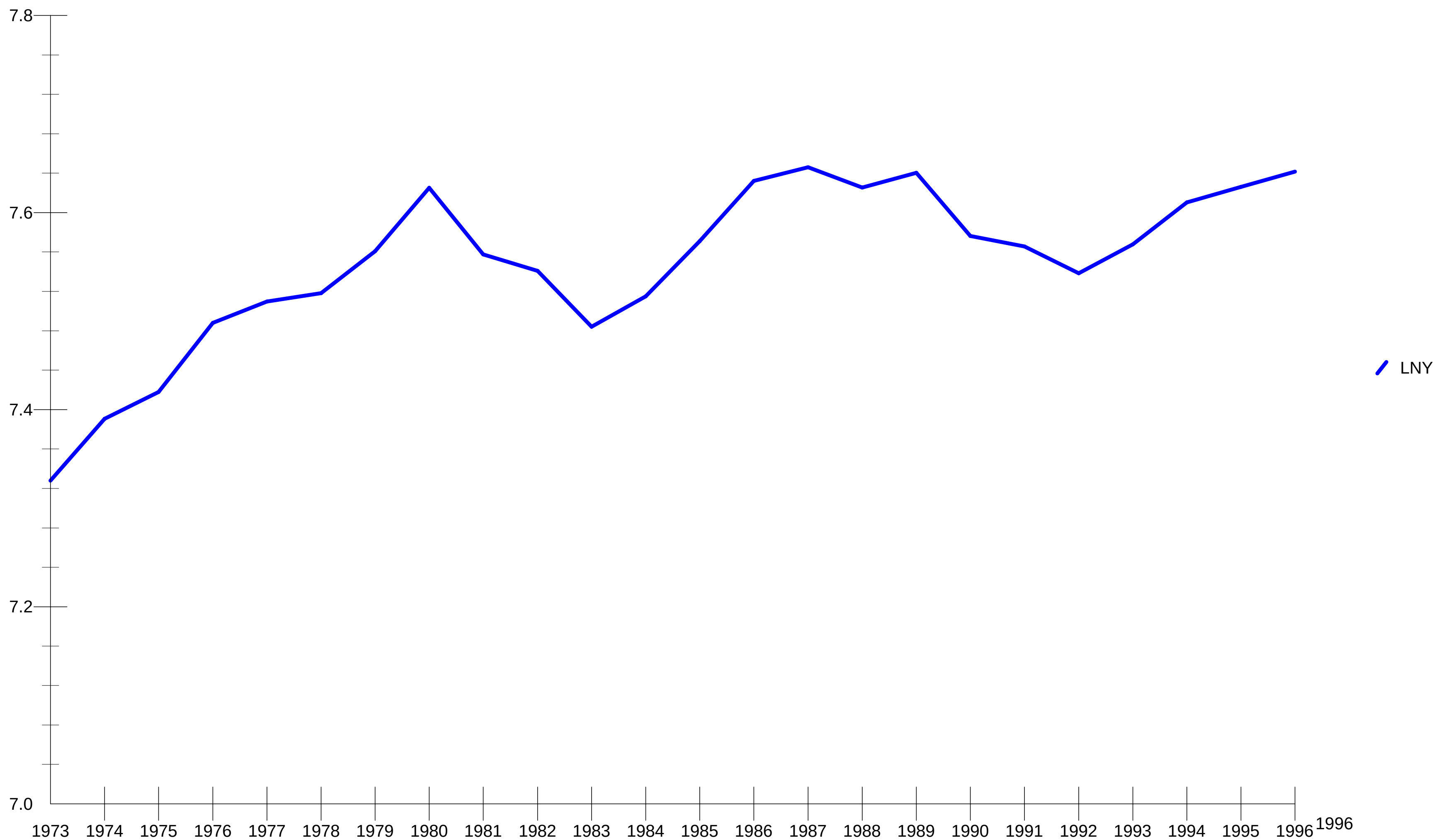
  for the case given by equations 13 and 14, though the γ parameter would have to be explicitly solved for from $\widehat{-\beta_{12}}$ and $\widehat{-\beta_{22}}$.

- For the case given by equations 16 and 17, we specify:

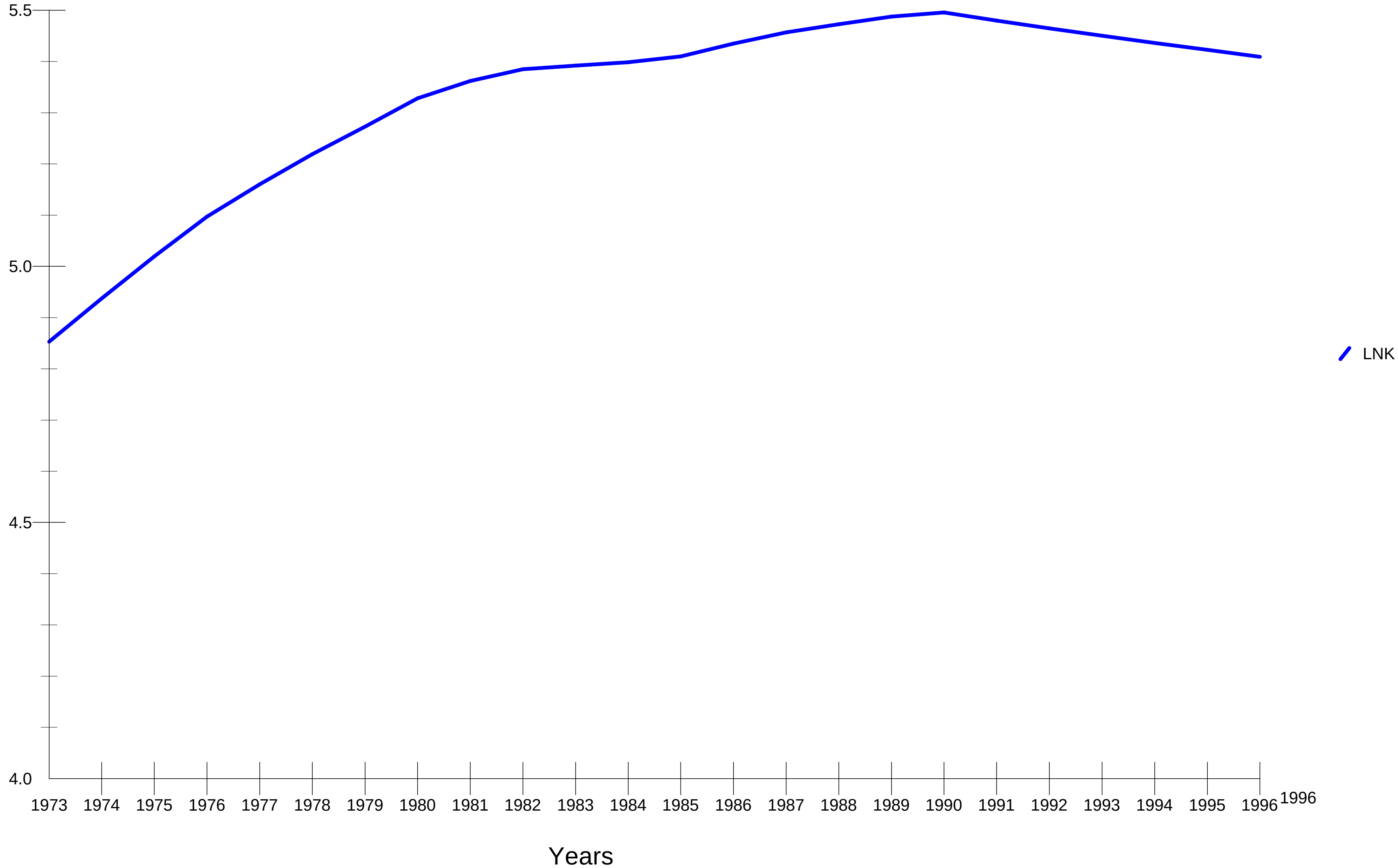
$$\Pi z_{t-k+1} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{bmatrix} \begin{bmatrix} 1 & -\beta_{12} & 0 \\ -\beta_{21} & 0 & 1 \end{bmatrix} \begin{bmatrix} y \\ k \\ r \end{bmatrix}_{t-k+1}$$

where $\widehat{\beta_{21}}$ provides a direct estimate of the γ parameter.

- Consider: Brazil.
- ln(Y):

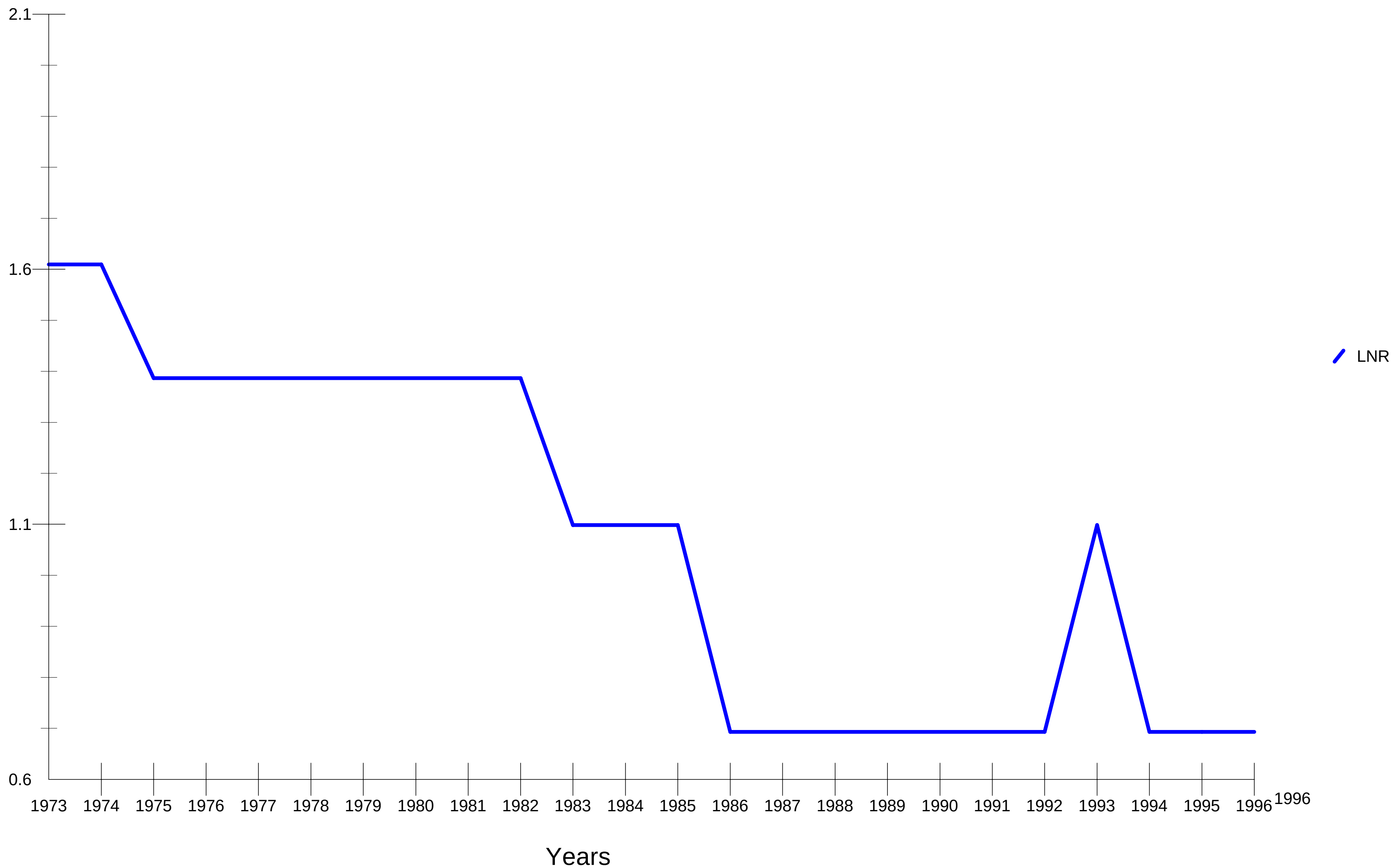


• $\ln(K/L)$:



5.5
5.0
4.5
4.0
1973
1974
1975
1976
1977
1978
1979
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1996
Years
LNK

- ln(R):



2.1
1.6
1.1
0.6
1973
1974
1975
1976
1977
1978
1979
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1996
Years
LNR

- Now estimate unrestricted VAR, to determine augmentation:

$$y_t = \alpha_0 + \sum \alpha_i y_{t-i} + \sum \alpha_j k_{t-j} + \sum \alpha_m r_{t-m} + \varepsilon_{1,t}$$
$$k_t = \beta_0 + \sum \beta_i y_{t-i} + \sum \beta_j k_{t-j} + \sum \beta_m r_{t-m} + \varepsilon_{2,t}$$
$$r_t = \gamma_0 + \sum \gamma_i y_{t-i} + \sum \gamma_j k_{t-j} + \sum \gamma_m r_{t-m} + \varepsilon_{3,t}$$

- Gives:

```
Test Statistics and Choice Criteria for Selecting the Order of the VAR Model
******************************************************************************
 Based on 21 observations from 1976 to 1996. Order of VAR = 3
 List of variables included in the unrestricted VAR:
 LNY             LNK             LNR
 List of deterministic and/or exogenous variables:
 C
******************************************************************************
 Order    LL        AIC      SBC            LR test         Adjusted LR test
   3   146.0612  116.0612  100.3934            ------               ------
   2   139.1175  118.1175  107.1500  CHSQ(  9)=  13.8875[.126]    7.2744[.609]
   1   119.7215  107.7215  101.4543  CHSQ( 18)=  52.6795[.000]   27.5940[.069]
   0    59.7050   56.7050   55.1382  CHSQ( 27)= 172.7124[.000]   90.4684[.000]
******************************************************************************
 AIC=Akaike Information Criterion     SBC=Schwarz Bayesian Criterion
```

$\Longrightarrow$ VAR(2)

- Now test for cointegration, by Pantula:

```
            Cointegration with no intercepts or trends in the VAR
  Cointegration LR Test Based on Maximal Eigenvalue of the Stochastic Matrix
*******************************************************************************
 22 observations from 1975 to 1996. Order of VAR = 2.
 List of variables included in the cointegrating vector:
 LNY            LNK            LNR
 List of eigenvalues in descending order:
.33661     .17543    .086331
*******************************************************************************
 Null    Alternative    Statistic     95% Critical Value     90%Critical Value
 r = 0      r = 1         9.0286           17.6800               15.5700
 r<= 1      r = 2         4.2436           11.0300                9.2800
 r<= 2      r = 3         1.9863            4.1600                3.0400
*******************************************************************************
 Use the above table to determine r (the number of cointegratingvectors).
```

```
            Cointegration with no intercepts or trends in the VAR
         Cointegration LR Test Based on Trace of the Stochastic Matrix
*******************************************************************************
 22 observations from 1975 to 1996. Order of VAR = 2.
 List of variables included in the cointegrating vector:
 LNY            LNK            LNR
 List of eigenvalues in descending order:
.33661     .17543    .086331
*******************************************************************************
 Null    Alternative    Statistic     95% Critical Value     90%Critical Value
 r = 0      r>= 1        15.2585           24.0500               21.4600
 r<= 1      r>= 2         6.2299           12.3600               10.2500
 r<= 2      r = 3         1.9863            4.1600                3.0400
*******************************************************************************
 Use the above table to determine r (the number of cointegratingvectors).
```

- And:

```
      Cointegration with restricted intercepts and no trends in the VAR
   Cointegration LR Test Based on Maximal Eigenvalue of the Stochastic Matrix
*******************************************************************************
 22 observations from 1975 to 1996. Order of VAR = 2.
 List of variables included in the cointegrating vector:
 LNY             LNK             LNR             Intercept
 List of eigenvalues in descending order:
.38699     .27025     .14632       0.00
*******************************************************************************
 Null    Alternative    Statistic     95% Critical Value     90%Critical Value
 r = 0      r = 1        10.7662           22.0400               19.8600
 r<= 1      r = 2         6.9310           15.8700               13.8100
 r<= 2      r = 3         3.4804            9.1600                7.5300
*******************************************************************************
 Use the above table to determine r (the number of cointegratingvectors).
```

```
      Cointegration with restricted intercepts and no trends in the VAR
         Cointegration LR Test Based on Trace of the Stochastic Matrix
*******************************************************************************
 22 observations from 1975 to 1996. Order of VAR = 2.
 List of variables included in the cointegrating vector:
 LNY             LNK             LNR             Intercept
 List of eigenvalues in descending order:
.38699     .27025     .14632       0.00
*******************************************************************************
 Null    Alternative    Statistic     95% Critical Value     90%Critical Value
 r = 0      r>= 1        21.1776           34.8700               31.9300
 r<= 1      r>= 2        10.4115           20.1800               17.8800
 r<= 2      r = 3         3.4804            9.1600                7.5300
*******************************************************************************
 Use the above table to determine r (the number of cointegratingvectors).
```

- To:

```
     Cointegration with unrestricted intercepts and no trends in the VAR
   Cointegration LR Test Based on Maximal Eigenvalue of the Stochastic Matrix
*******************************************************************************
 22 observations from 1975 to 1996. Order of VAR = 2.
 List of variables included in the cointegrating vector:
 LNY             LNK             LNR
 List of eigenvalues in descending order:
.34677     .18444   .0053661
*******************************************************************************
 Null    Alternative    Statistic     95% Critical Value     90%Critical Value
 r = 0      r = 1         9.3680           21.1200                19.0200
 r<= 1      r = 2         4.4852           14.8800                12.9800
 r<= 2      r = 3         .11837            8.0700                 6.5000
*******************************************************************************
 Use the above table to determine r (the number of cointegratingvectors).


     Cointegration with unrestricted intercepts and no trends in the VAR
         Cointegration LR Test Based on Trace of the Stochastic Matrix
*******************************************************************************
 22 observations from 1975 to 1996. Order of VAR = 2.
 List of variables included in the cointegrating vector:
 LNY             LNK             LNR
 List of eigenvalues in descending order:
.34677     .18444   .0053661
*******************************************************************************
 Null    Alternative    Statistic     95% Critical Value     90%Critical Value
 r = 0      r>= 1        13.9716           31.5400                28.7800
 r<= 1      r>= 2         4.6036           17.8600                15.7500
 r<= 2      r = 3         .11837            8.0700                 6.5000
*******************************************************************************
 Use the above table to determine r (the number of cointegratingvectors).
```

- And:

```
  Cointegration with unrestricted intercepts and restricted trendsin the VAR
  Cointegration LR Test Based on Maximal Eigenvalue of the Stochastic Matrix
******************************************************************************
 22 observations from 1975 to 1996. Order of VAR = 2.
 List of variables included in the cointegrating vector:
 LNY            LNK            LNR            Trend
 List of eigenvalues in descending order:
.55738     .33969     .17685       0.00
******************************************************************************
 Null    Alternative    Statistic     95% Critical Value     90%Critical Value
 r = 0      r = 1        17.9311          25.4200               23.1000
 r<= 1      r = 2         9.1309          19.2200               17.1800
 r<= 2      r = 3         4.2817          12.3900               10.5500
******************************************************************************
 Use the above table to determine r (the number of cointegratingvectors).
```

```
  Cointegration with unrestricted intercepts and restricted trendsin the VAR
         Cointegration LR Test Based on Trace of the Stochastic Matrix
******************************************************************************
 22 observations from 1975 to 1996. Order of VAR = 2.
 List of variables included in the cointegrating vector:
 LNY            LNK            LNR            Trend
 List of eigenvalues in descending order:
.55738     .33969     .17685       0.00
******************************************************************************
 Null    Alternative    Statistic     95% Critical Value     90%Critical Value
 r = 0      r>= 1        31.3437          42.3400               39.3400
 r<= 1      r>= 2        13.4126          25.7700               23.0800
 r<= 2      r = 3         4.2817          12.3900               10.5500
******************************************************************************
 Use the above table to determine r (the number of cointegratingvectors).
```

- To finally:

```
Cointegration with unrestricted intercepts and unrestricted trends in the VAR
   Cointegration LR Test Based on Maximal Eigenvalue of the Stochastic Matrix
******************************************************************************
 22 observations from 1975 to 1996. Order of VAR = 2.
 List of variables included in the cointegrating vector:
 LNY             LNK             LNR
 List of eigenvalues in descending order:
.52299     .28627    .041511
******************************************************************************
 Null    Alternative    Statistic     95% Critical Value     90%Critical Value
 r = 0      r = 1        16.2850           24.3500               22.2600
 r<= 1      r = 2         7.4196           18.3300               16.2800
 r<= 2      r = 3         .93274           11.5400                9.7500
******************************************************************************
 Use the above table to determine r (the number of cointegratingvectors).
```

```
 Cointegration with unrestricted intercepts and unrestricted trends in the VAR
          Cointegration LR Test Based on Trace of the Stochastic Matrix
******************************************************************************
 22 observations from 1975 to 1996. Order of VAR = 2.
 List of variables included in the cointegrating vector:
 LNY             LNK             LNR
 List of eigenvalues in descending order:
.52299     .28627    .041511
******************************************************************************
 Null    Alternative    Statistic     95% Critical Value     90%Critical Value
 r = 0      r>= 1        24.6373           39.3300               36.2800
 r<= 1      r>= 2         8.3524           23.8300               21.2300
 r<= 2      r = 3         .93274           11.5400                9.7500
******************************************************************************
 Use the above table to determine r (the number of cointegratingvectors).
```

$\Longrightarrow$ no CV's.

- But consider unrestricted VAR(4):

```
Test Statistics and Choice Criteria for Selecting the Order of the VAR Model
*******************************************************************************
 Based on 20 observations from 1977 to 1996. Order of VAR = 4
 List of variables included in the unrestricted VAR:
 LNY             LNK             LNR
 List of deterministic and/or exogenous variables:
 C
*******************************************************************************
 Order    LL        AIC      SBC             LR test         Adjusted LR test
   4   159.0019  120.0019  100.5851              ------                ------
   3   139.5931  109.5931   94.6571  CHSQ(  9)=  38.8175[.000]   13.5861[.138]
   2   132.0637  111.0637  100.6086  CHSQ( 18)=  53.8763[.000]   18.8567[.401]
   1   113.3363  101.3363   95.3619  CHSQ( 27)=  91.3312[.000]   31.9659[.233]
   0    61.6795   58.6795   57.1859  CHSQ( 36)= 194.6448[.000]   68.1257[.001]
*******************************************************************************
 AIC=Akaike Information Criterion     SBC=Schwarz Bayesian Criterion
```

- Now Pantula:

```
            Cointegration with no intercepts or trends in the VAR
   Cointegration LR Test Based on Maximal Eigenvalue of the Stochastic Matrix
******************************************************************************
 20 observations from 1977 to 1996. Order of VAR = 4.
 List of variables included in the cointegrating vector:
 LNY             LNK             LNR
 List of eigenvalues in descending order:
.85334     .40092     .30404
******************************************************************************
 Null    Alternative    Statistic     95% Critical Value     90%Critical Value
 r = 0      r = 1        38.3925          17.6800                15.5700
 r<= 1      r = 2        10.2473          11.0300                 9.2800
 r<= 2      r = 3         7.2493           4.1600                 3.0400
******************************************************************************
 Use the above table to determine r (the number of cointegratingvectors).
```

```
            Cointegration with no intercepts or trends in the VAR
         Cointegration LR Test Based on Trace of the Stochastic Matrix
******************************************************************************
 20 observations from 1977 to 1996. Order of VAR = 4.
 List of variables included in the cointegrating vector:
 LNY             LNK             LNR
 List of eigenvalues in descending order:
.85334     .40092     .30404
******************************************************************************
 Null    Alternative    Statistic     95% Critical Value     90%Critical Value
 r = 0      r>= 1        55.8892          24.0500                21.4600
 r<= 1      r>= 2        17.4967          12.3600                10.2500
 r<= 2      r = 3         7.2493           4.1600                 3.0400
******************************************************************************
 Use the above table to determine r (the number of cointegratingvectors).
```

$$\Longrightarrow r = 2$$

- And:

```
      Cointegration with restricted intercepts and no trends in the VAR
   Cointegration LR Test Based on Maximal Eigenvalue of the Stochastic Matrix
******************************************************************************
 20 observations from 1977 to 1996. Order of VAR = 4.
 List of variables included in the cointegrating vector:
 LNY             LNK             LNR             Intercept
 List of eigenvalues in descending order:
.87559     .45751     .37185      .0000
******************************************************************************
 Null    Alternative    Statistic     95% Critical Value     90%Critical Value
 r = 0      r = 1        41.6835           22.0400                19.8600
 r<= 1      r = 2        12.2315           15.8700                13.8100
 r<= 2      r = 3         9.2997            9.1600                 7.5300
******************************************************************************
 Use the above table to determine r (the number of cointegratingvectors).
```

```
      Cointegration with restricted intercepts and no trends in the VAR
         Cointegration LR Test Based on Trace of the Stochastic Matrix
******************************************************************************
 20 observations from 1977 to 1996. Order of VAR = 4.
 List of variables included in the cointegrating vector:
 LNY             LNK             LNR             Intercept
 List of eigenvalues in descending order:
.87559     .45751     .37185      .0000
******************************************************************************
 Null    Alternative    Statistic     95% Critical Value     90%Critical Value
 r = 0      r>= 1        63.2147           34.8700                31.9300
 r<= 1      r>= 2        21.5312           20.1800                17.8800
 r<= 2      r = 3         9.2997            9.1600                 7.5300
******************************************************************************
 Use the above table to determine r (the number of cointegratingvectors).
```

- To:

```
     Cointegration with unrestricted intercepts and no trends in the VAR
   Cointegration LR Test Based on Maximal Eigenvalue of the Stochastic Matrix
******************************************************************************
 20 observations from 1977 to 1996. Order of VAR = 4.
 List of variables included in the cointegrating vector:
 LNY             LNK             LNR
 List of eigenvalues in descending order:
.82267     .45192   .4581E-3
******************************************************************************
 Null    Alternative    Statistic     95% Critical Value     90%Critical Value
 r = 0      r = 1        34.5948           21.1200                19.0200
 r<= 1      r = 2        12.0267           14.8800                12.9800
 r<= 2      r = 3       .0091643            8.0700                 6.5000
******************************************************************************
 Use the above table to determine r (the number of cointegratingvectors).
```

```
     Cointegration with unrestricted intercepts and no trends in the VAR
         Cointegration LR Test Based on Trace of the Stochastic Matrix
******************************************************************************
 20 observations from 1977 to 1996. Order of VAR = 4.
 List of variables included in the cointegrating vector:
 LNY             LNK             LNR
 List of eigenvalues in descending order:
.82267     .45192   .4581E-3
******************************************************************************
 Null    Alternative    Statistic     95% Critical Value     90%Critical Value
 r = 0      r>= 1        46.6307           31.5400                28.7800
 r<= 1      r>= 2        12.0359           17.8600                15.7500
 r<= 2      r = 3       .0091643            8.0700                 6.5000
******************************************************************************
 Use the above table to determine r (the number of cointegratingvectors).
```

- And:

```
   Cointegration with unrestricted intercepts and restricted trendsin the VAR
   Cointegration LR Test Based on Maximal Eigenvalue of the Stochastic Matrix
*******************************************************************************
 20 observations from 1977 to 1996. Order of VAR = 4.
 List of variables included in the cointegrating vector:
 LNY             LNK             LNR             Trend
 List of eigenvalues in descending order:
.92718     .81262     .41765       0.00
*******************************************************************************
 Null    Alternative    Statistic     95% Critical Value     90%Critical Value
 r = 0      r = 1        52.3948           25.4200                23.1000
 r<= 1      r = 2        33.4919           19.2200                17.1800
 r<= 2      r = 3        10.8137           12.3900                10.5500
*******************************************************************************
 Use the above table to determine r (the number of cointegratingvectors).
```

```
   Cointegration with unrestricted intercepts and restricted trendsin the VAR
          Cointegration LR Test Based on Trace of the Stochastic Matrix
*******************************************************************************
 20 observations from 1977 to 1996. Order of VAR = 4.
 List of variables included in the cointegrating vector:
 LNY             LNK             LNR             Trend
 List of eigenvalues in descending order:
.92718     .81262     .41765       0.00
*******************************************************************************
 Null    Alternative    Statistic     95% Critical Value     90%Critical Value
 r = 0      r>= 1        96.7004           42.3400                39.3400
 r<= 1      r>= 2        44.3056           25.7700                23.0800
 r<= 2      r = 3        10.8137           12.3900                10.5500
*******************************************************************************
 Use the above table to determine r (the number of cointegratingvectors).
```

- And finally:

```
Cointegration with unrestricted intercepts and unrestricted trends in the VAR
   Cointegration LR Test Based on Maximal Eigenvalue of the Stochastic Matrix
*******************************************************************************
 20 observations from 1977 to 1996. Order of VAR = 4.
 List of variables included in the cointegrating vector:
 LNY             LNK             LNR
 List of eigenvalues in descending order:
.90764     .74435     .23656
*******************************************************************************
 Null    Alternative    Statistic     95% Critical Value     90%Critical Value
 r = 0      r = 1        47.6422           24.3500               22.2600
 r<= 1      r = 2        27.2793           18.3300               16.2800
 r<= 2      r = 3         5.3983           11.5400                9.7500
*******************************************************************************
 Use the above table to determine r (the number of cointegratingvectors).
```

```
 Cointegration with unrestricted intercepts and unrestricted trends in the VAR
          Cointegration LR Test Based on Trace of the Stochastic Matrix
*******************************************************************************
 20 observations from 1977 to 1996. Order of VAR = 4.
 List of variables included in the cointegrating vector:
 LNY             LNK             LNR
 List of eigenvalues in descending order:
.90764     .74435     .23656
*******************************************************************************
 Null    Alternative    Statistic     95% Critical Value     90%Critical Value
 r = 0      r>= 1        80.3198           39.3300               36.2800
 r<= 1      r>= 2        32.6776           23.8300               21.2300
 r<= 2      r = 3         5.3983           11.5400                9.7500
*******************************************************************************
 Use the above table to determine r (the number of cointegratingvectors).
```

- Proceed by:
  - unrestricted intercepts
  - restricted trends
  - theory: unlikely to be full specification; trends to cover additional explanatory dimensions
    $\Longrightarrow r = 2$
    $\Longrightarrow r^2 = 4$ just identifying restrictions

- Now estimate:

$$\Pi z_{t-k+1} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{bmatrix} \begin{bmatrix} 1 & -\beta_{12} & 0 \\ 0 & -\beta_{22} & 1 \end{bmatrix} \begin{bmatrix} y \\ k \\ r \end{bmatrix}_{t-k+1}$$

- To obtain:

```
        ML estimates subject to exactly identifying restriction(s)
    Estimates of Restricted Cointegrating Relations (SE's in Brackets)
                       Converged after 2 iterations
 Cointegration with unrestricted intercepts and restricted trendsin the VAR
*******************************************************************************
 20 observations from 1977 to 1996. Order of VAR = 4, chosen r =2.
 List of variables included in the cointegrating vector:
 LNY             LNK             LNR             Trend
*******************************************************************************
 List of imposed restriction(s) on cointegrating vectors:
 a1=1;a3=0;  b1=0;b3=1;
*******************************************************************************
                  Vector  1      Vector  2
 LNY                  1.0000           0.00
                  (   *NONE*)    (   *NONE*)

 LNK                 -1.4336        -4.7517
                  (   .49943)    (   3.5235)

 LNR                    0.00         1.0000
                  (   *NONE*)    (   *NONE*)

 Trend                -.022666      -.0030977
                  ( .0096154)    (  .067737)

*******************************************************************************
 LL subject to exactly identifying restrictions= 178.6299
*******************************************************************************
```

- With:

```
      ECM for variable LNY estimated by OLS based on cointegrating VAR(4)
*******************************************************************************
 Dependent variable is dLNY
 20 observations used for estimation from 1977 to 1996
*******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 Intercept                 -1.9050            3.9123             -.48692[.639]
 dLNY1                     -1.4776            .99030             -1.4921[.174]
 dLNK1                     -.49077            2.7194             -.18047[.861]
 dLNR1                      .11663            .17378              .67115[.521]
 dLNY2                     -.70310            .60425             -1.1636[.278]
 dLNK2                     -3.1199            2.2767             -1.3704[.208]
 dLNR2                     .065539            .14161              .46282[.656]
 dLNY3                     -.21946            .30151             -.72787[.487]
 dLNK3                     -3.8471            2.5847             -1.4884[.175]
 dLNR3                    -.037545            .10701             -.35087[.735]
 ecm1(-1)                   1.3358            1.2320              1.0842[.310]
 ecm2(-1)                  -.10690            .18290             -.58445[.575]
*******************************************************************************
 List of additional temporary variables created:
 dLNY = LNY-LNY(-1)
 dLNY1 = LNY(-1)-LNY(-2)
 dLNK1 = LNK(-1)-LNK(-2)
 dLNR1 = LNR(-1)-LNR(-2)
 dLNY2 = LNY(-2)-LNY(-3)
 dLNK2 = LNK(-2)-LNK(-3)
 dLNR2 = LNR(-2)-LNR(-3)
 dLNY3 = LNY(-3)-LNY(-4)
 dLNK3 = LNK(-3)-LNK(-4)
 dLNR3 = LNR(-3)-LNR(-4)
 ecm1 =    1.0000*LNY   -1.4336*LNK      0.00*LNR  -.022666*Trend;ecm2 =
0.00*LNY   -4.7517*LNK +   1.0000*LNR -.0030977*Trend
*******************************************************************************
 R-Squared                     .63698   R-Bar-Squared                   .13782
 S.E. of Regression            .036993   F-stat.    F( 11,   8)    1.2761[.373]
 Mean of Dependent Variable  .0076881   S.D. of Dependent Variable     .039840
 Residual Sum of Squares       .010948   Equation Log-likelihood        46.7246
 Akaike Info. Criterion        34.7246   Schwarz Bayesian Criterion     28.7502
 DW-statistic                   1.9986   System Log-likelihood         178.6299
*******************************************************************************


                               Diagnostic Tests
*******************************************************************************
*    Test Statistics  *        LM Version        *         F Version          *
*******************************************************************************
*                     *                          *                            *
* A:Serial Correlation*CHSQ(   1)=  .020188[.887]*F(   1,   7)= .0070729[.935]*
*                     *                          *                            *
* B:Functional Form   *CHSQ(   1)=   1.8872[.170]*F(   1,   7)=   .72934[.421]*
*                     *                          *                            *
* C:Normality         *CHSQ(   2)=  .015115[.992]*       Not applicable       *
*                     *                          *                            *
* D:Heteroscedasticity*CHSQ(   1)=   .61524[.433]*F(   1,  18)=   .57129[.460]*
*******************************************************************************
   A:Lagrange multiplier test of residual serial correlation
   B:Ramsey's RESET test using the square of the fitted values
   C:Based on a test of skewness and kurtosis of residuals
```

- And:

```
      ECM for variable LNR estimated by OLS based on cointegrating VAR(4)
*******************************************************************************
 Dependent variable is dLNR
 20 observations used for estimation from 1977 to 1996
*******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 Intercept                   1.5627              16.0909            .097114[.925]
 dLNY1                       3.9892               4.0730             .97942[.356]
 dLNK1                      -2.8697              11.1846            -.25657[.804]
 dLNR1                      -.50032               .71475            -.70000[.504]
 dLNY2                       4.7931               2.4852             1.9286[.090]
 dLNK2                      -5.2192               9.3639            -.55737[.593]
 dLNR2                      -.45113               .58243            -.77457[.461]
 dLNY3                      -.52381               1.2401            -.42240[.684]
 dLNK3                      25.6764              10.6309             2.4153[.042]
 dLNR3                       .18629               .44010             .42329[.683]
 ecm1(-1)                   -2.9661               5.0671            -.58536[.574]
 ecm2(-1)                    .13686               .75226             .18194[.860]
*******************************************************************************
 List of additional temporary variables created:
 dLNR = LNR-LNR(-1)
 dLNY1 = LNY(-1)-LNY(-2)
 dLNK1 = LNK(-1)-LNK(-2)
 dLNR1 = LNR(-1)-LNR(-2)
 dLNY2 = LNY(-2)-LNY(-3)
 dLNK2 = LNK(-2)-LNK(-3)
 dLNR2 = LNR(-2)-LNR(-3)
 dLNY3 = LNY(-3)-LNY(-4)
 dLNK3 = LNK(-3)-LNK(-4)
 dLNR3 = LNR(-3)-LNR(-4)
 ecm1 =    1.0000*LNY   -1.4336*LNK      0.00*LNR  -.022666*Trend;ecm2 =
0.00*LNY   -4.7517*LNK +   1.0000*LNR -.0030977*Trend
*******************************************************************************
 R-Squared                     .66447   R-Bar-Squared                     .20311
 S.E. of Regression            .15215   F-stat.    F( 11,   8)    1.4402[.309]
 Mean of Dependent Variable  -.034657   S.D. of Dependent Variable       .17044
 Residual Sum of Squares       .18520   Equation Log-likelihood         18.4419
 Akaike Info. Criterion        6.4419   Schwarz Bayesian Criterion       .46755
 DW-statistic                  2.0858   System Log-likelihood          178.6299
*******************************************************************************


                               Diagnostic Tests
*******************************************************************************
*    Test Statistics  *        LM Version        *         F Version          *
*******************************************************************************
*                     *                          *                            *
* A:Serial Correlation*CHSQ(   1)=   .80882[.368]*F(   1,   7)=   .29502[.604]*
*                     *                          *                            *
* B:Functional Form   *CHSQ(   1)=   3.4994[.061]*F(   1,   7)=   1.4845[.263]*
*                     *                          *                            *
* C:Normality         *CHSQ(   2)=   2.2545[.324]*       Not applicable       *
*                     *                          *                            *
* D:Heteroscedasticity*CHSQ(   1)=   .15419[.695]*F(   1,  18)=   .13985[.713]*
*******************************************************************************
   A:Lagrange multiplier test of residual serial correlation
   B:Ramsey's RESET test using the square of the fitted values
   C:Based on a test of skewness and kurtosis of residuals
```

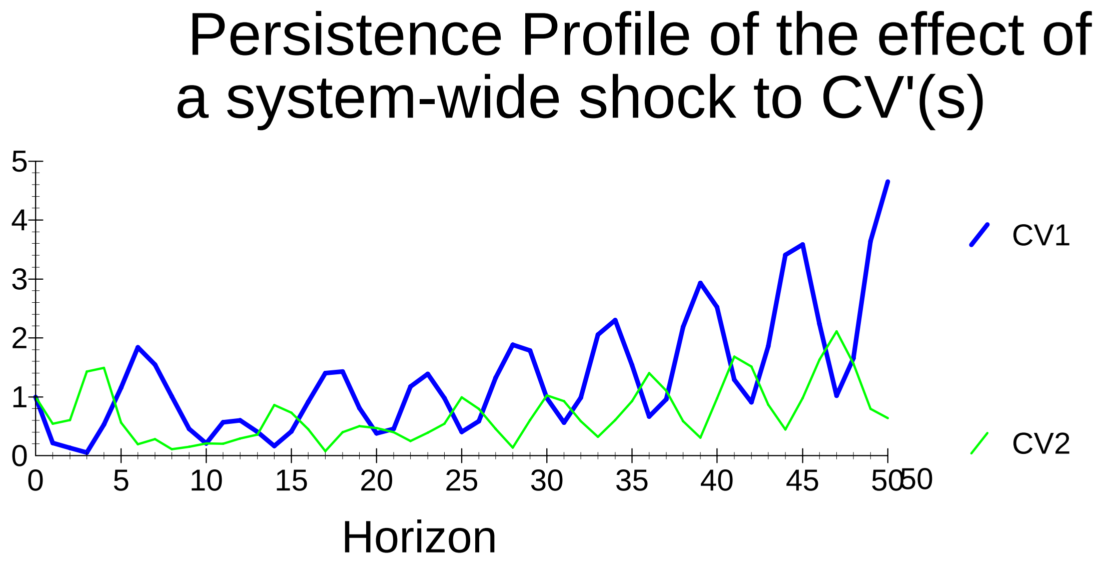
- And testing for "significance" of β_{12}:

```
          ML estimates subject to over identifying restriction(s)
      Estimates of Restricted Cointegrating Relations (SE's in Brackets)
                       Converged after 19 iterations
  Cointegration with unrestricted intercepts and restricted trendsin the VAR
*******************************************************************************
 20 observations from 1977 to 1996. Order of VAR = 4, chosen r =2.
 List of variables included in the cointegrating vector:
 LNY             LNK             LNR             Trend
*******************************************************************************
 List of imposed restriction(s) on cointegrating vectors:
 a1=1;a3=0;  b1=0;b3=1;  a2=0
*******************************************************************************
                  Vector  1      Vector  2
 LNY                  1.0000          .0000
                  (   *NONE*)    (   *NONE*)

 LNK                   .0000         5.4720
                  (   *NONE*)    (   .44301)

 LNR                    0.00         1.0000
                  (   *NONE*)    (   *NONE*)

 Trend               .0050437        .19463
                  ( .0066099)    (  .046515)

*******************************************************************************
 LR Test of Restrictions         CHSQ( 1)=   9.1886[.002]
 DF=Total no of restrictions(5) - no of just-identifying restrictions(4)
 LL subject to exactly identifying restrictions= 178.6299
 LL subject to over-identifying restrictions= 174.0356
*******************************************************************************
```

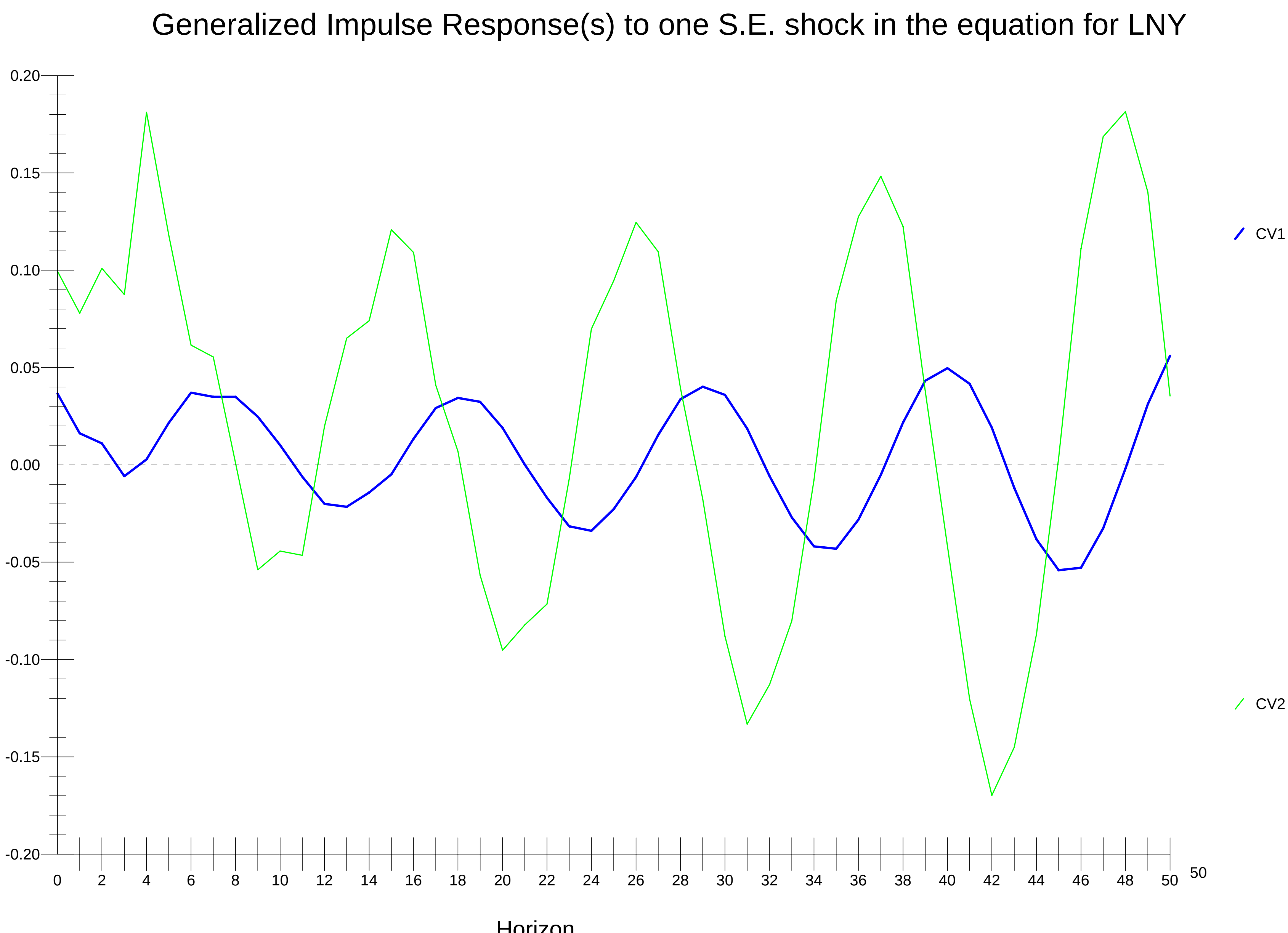
- And "significance" of β_{22}:

```
           ML estimates subject to over identifying restriction(s)
     Estimates of Restricted Cointegrating Relations (SE's in Brackets)
                       Converged after 507 iterations
 Cointegration with unrestricted intercepts and restricted trendsin the VAR
*******************************************************************************
 20 observations from 1977 to 1996. Order of VAR = 4, chosen r =2.
 List of variables included in the cointegrating vector:
 LNY             LNK             LNR             Trend
*******************************************************************************
 List of imposed restriction(s) on cointegrating vectors:
 a1=1;a3=0;  b1=0;b3=1;  b2=0
*******************************************************************************
                  Vector  1      Vector  2
 LNY                  1.0000         -.0000
                  (   *NONE*)    (   *NONE*)

 LNK                 -.77998          .0000
                  (  .028812)    (   *NONE*)

 LNR                   .0000         1.0000
                  (   *NONE*)    (   *NONE*)

 Trend              -.0016458         .14931
                  (   *NONE*)    (   *NONE*)

*******************************************************************************
 LR Test of Restrictions          CHSQ( 1)=  33.3352[.000]
 DF=Total no of restrictions(5) - no of just-identifying restrictions(4)
 LL subject to exactly identifying restrictions= 178.6299
 LL subject to over-identifying restrictions= 161.9623
*******************************************************************************
```

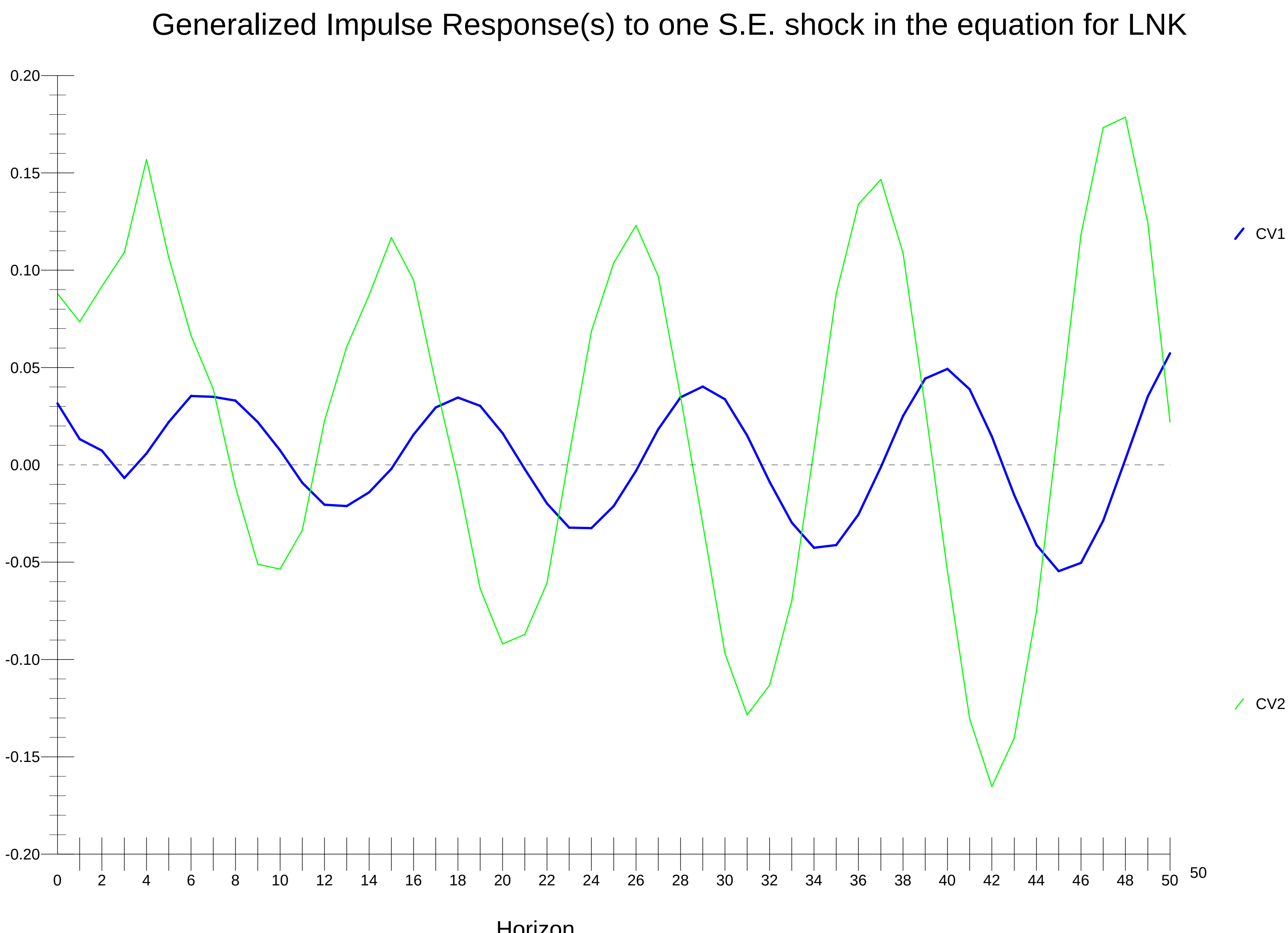
- And Persistence Profile of Shock to CV's:



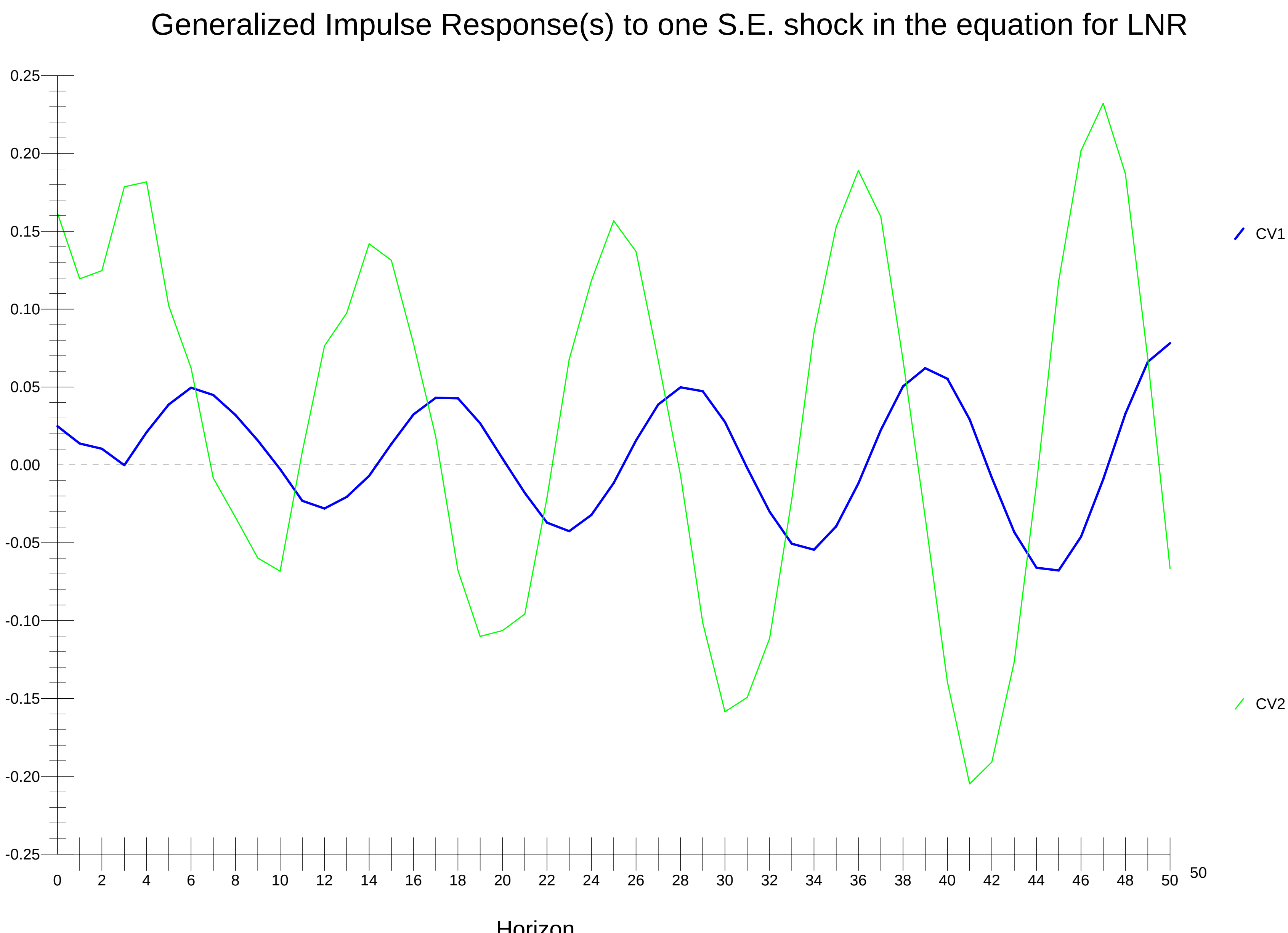
- Impulse response of CV's to shock in lnY equation:



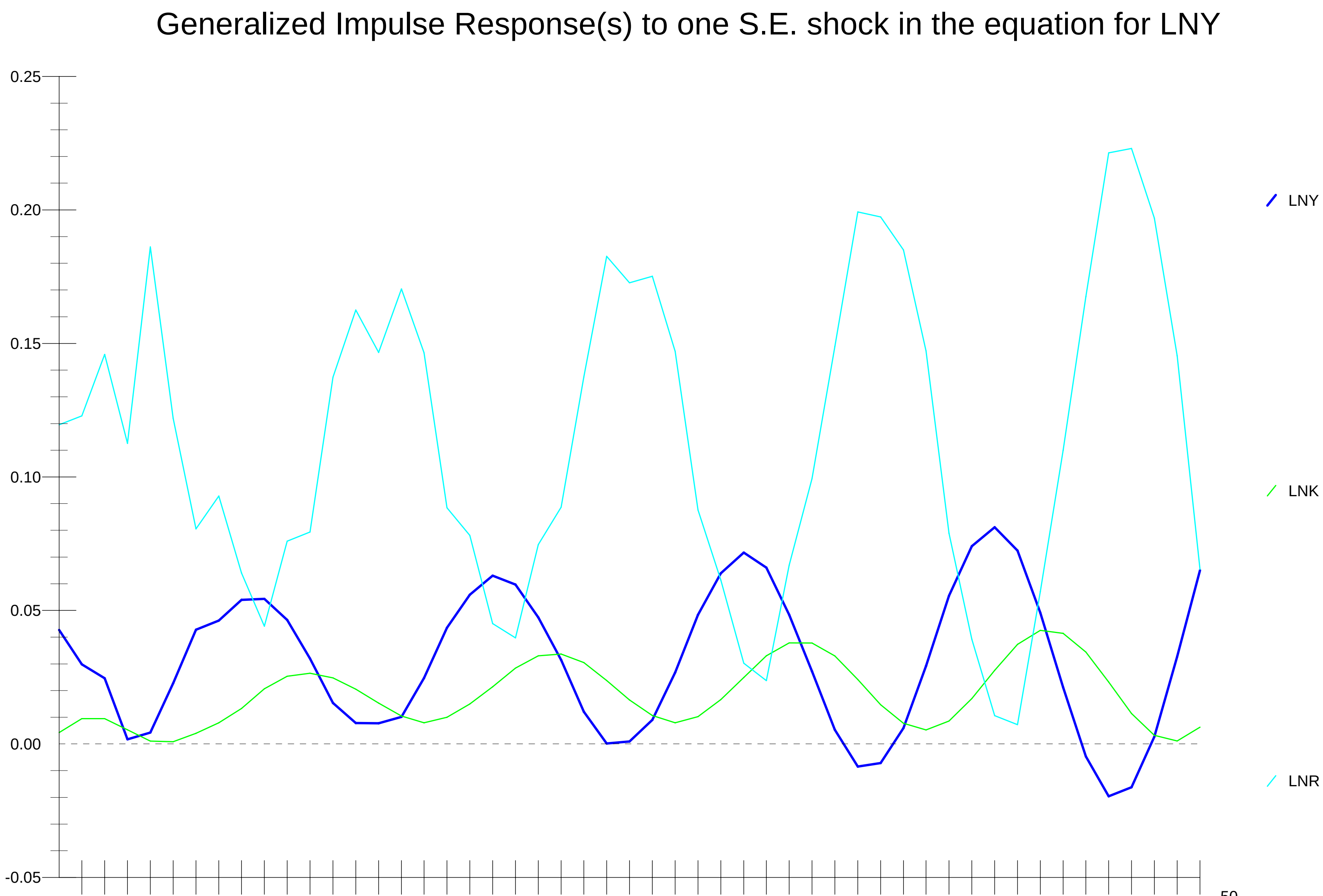
- Impulse response of CV's to shock in lnK equation:



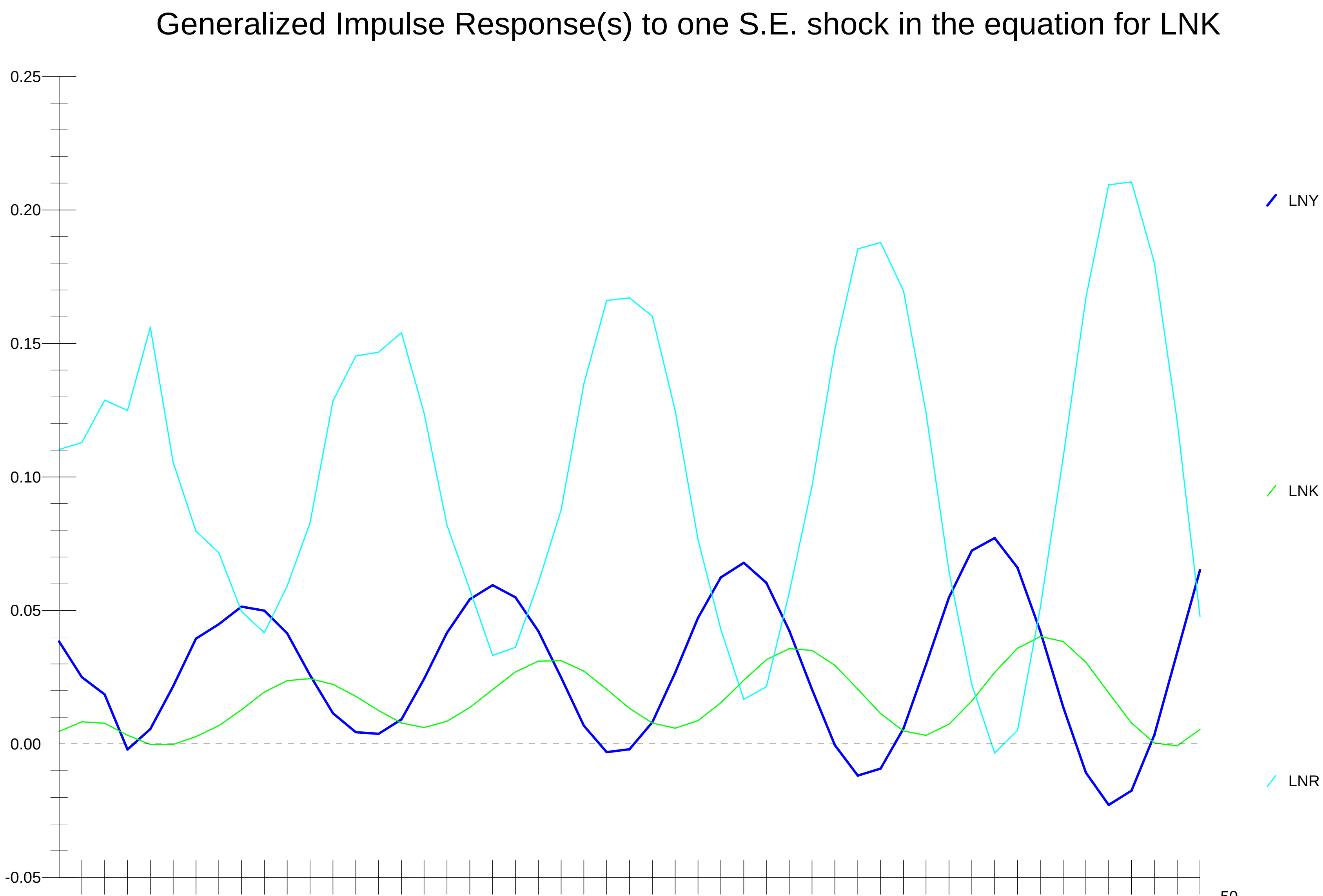
- Impulse response of CV's to shock in lnR equation:



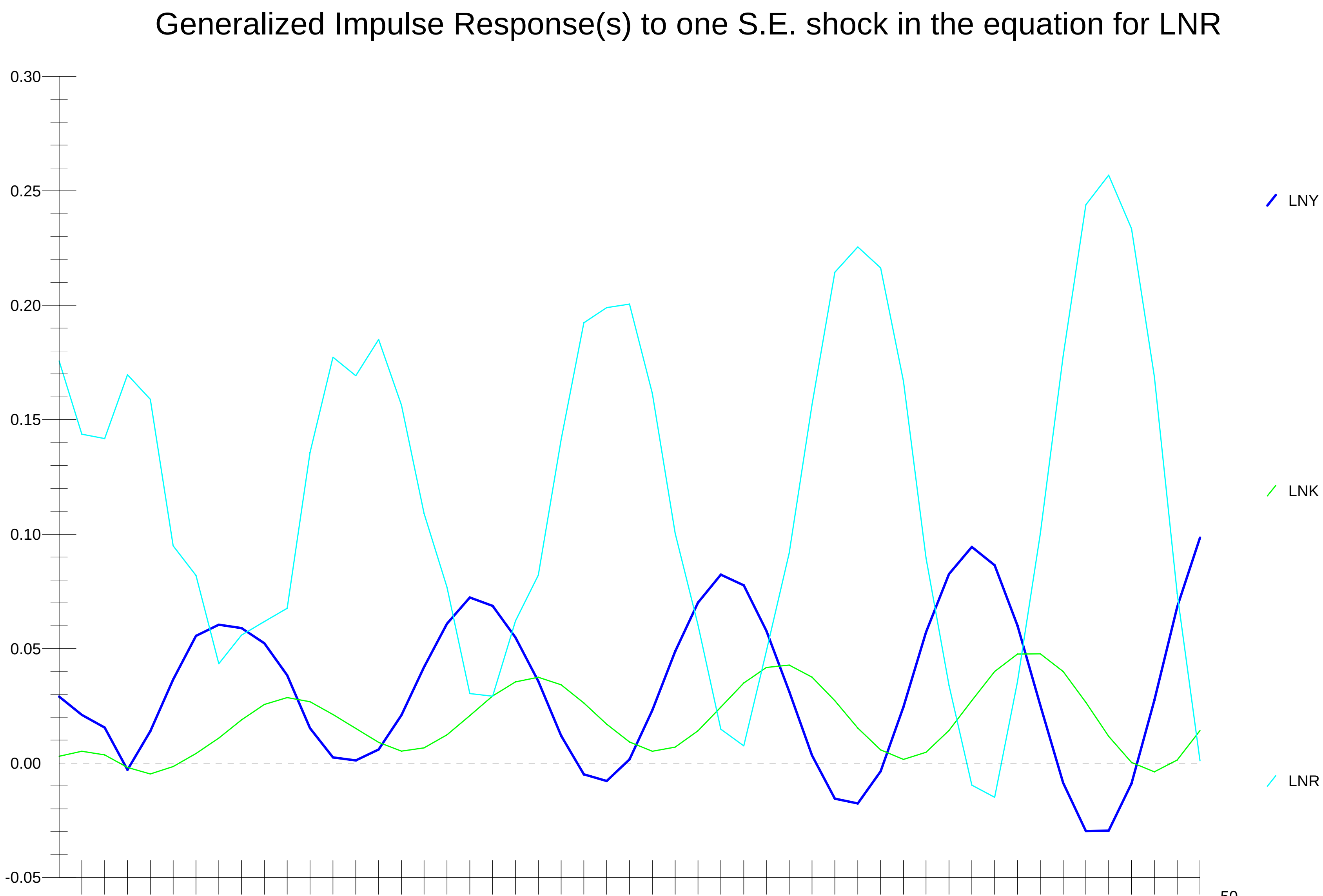
- Impulse response of lnY, lnK, lnR to shock in lnY equation:



- Impulse response of lnY, lnK, lnR to shock in lnK equation:



- Impulse response of lnY, lnK, lnR to shock in lnK equation:



- Alternatively specify:

$$\Pi z_{t-k+1} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{bmatrix} \begin{bmatrix} 1 & -\beta_{12} & 0 \\ -\beta_{21} & 0 & 1 \end{bmatrix} \begin{bmatrix} y \\ k \\ r \end{bmatrix}_{t-k+1}$$

- To obtain:

```
         ML estimates subject to exactly identifying restriction(s)
     Estimates of Restricted Cointegrating Relations (SE's in Brackets)
                         Converged after 2 iterations
  Cointegration with unrestricted intercepts and restricted trendsin the VAR
*******************************************************************************
 20 observations from 1977 to 1996. Order of VAR = 4, chosen r =2.
 List of variables included in the cointegrating vector:
 LNY             LNK             LNR             Trend
*******************************************************************************
 List of imposed restriction(s) on cointegrating vectors:
 a1=1;a3=0;    b2=0;b3=1;
*******************************************************************************
                  Vector  1      Vector  2
 LNY                  1.0000        -3.3146
                  (   *NONE*)    (   1.3159)

 LNK                 -1.4336           0.00
                  (   .49943)    (   *NONE*)

 LNR                    0.00         1.0000
                  (   *NONE*)    (   *NONE*)

 Trend              -.022666        .072030
                  ( .0096154)    (  .018815)

*******************************************************************************
 LL subject to exactly identifying restrictions= 178.6299
*******************************************************************************
```

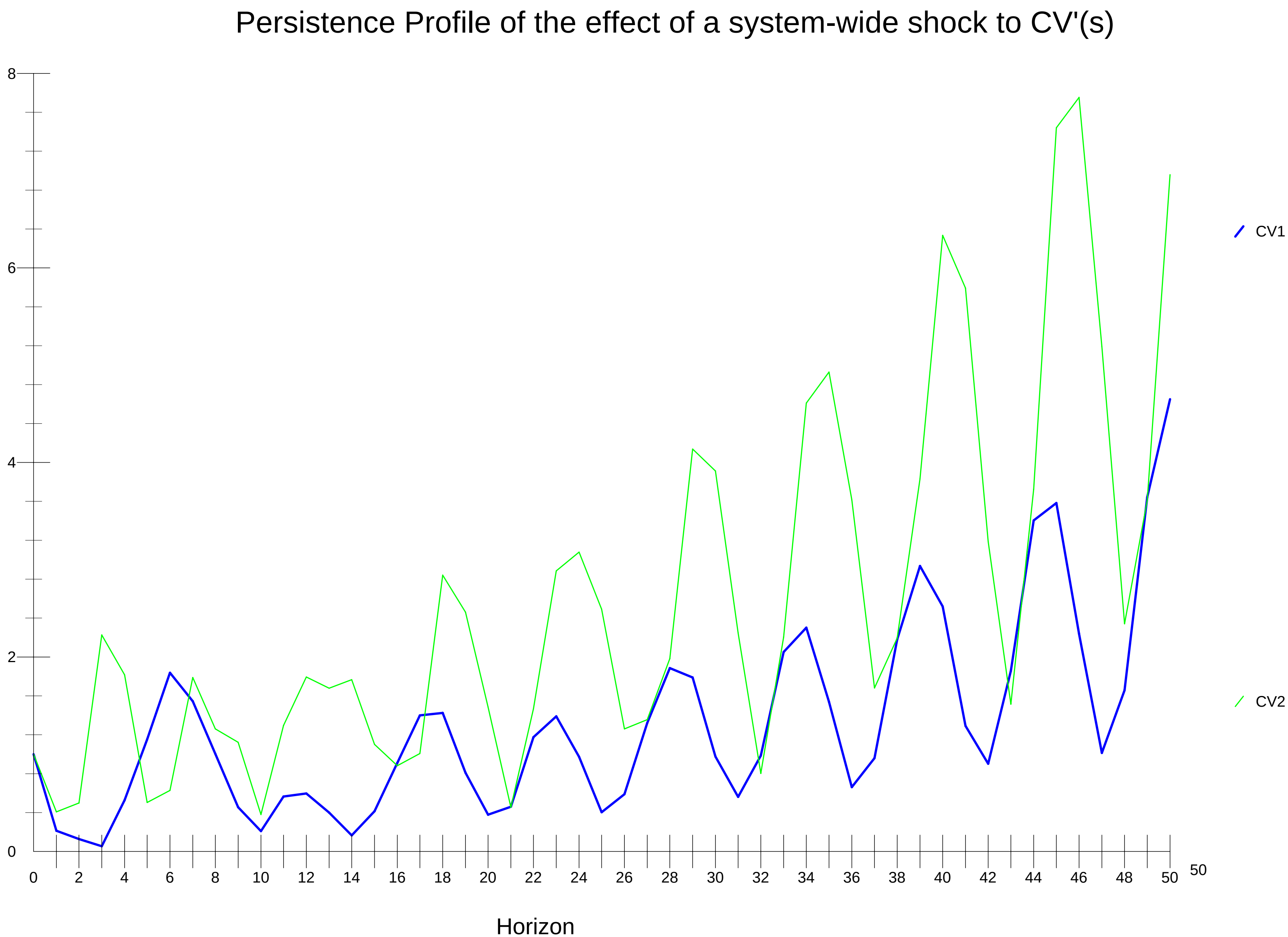
- with:

```
     ECM for variable LNY estimated by OLS based on cointegrating VAR(4)
******************************************************************************
 Dependent variable is dLNY
 20 observations used for estimation from 1977 to 1996
******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 Intercept                  -1.9050               3.9123          -.48692[.639]
 dLNY1                      -1.4776               .99030          -1.4921[.174]
 dLNK1                      -.49077               2.7194          -.18047[.861]
 dLNR1                       .11663               .17378           .67115[.521]
 dLNY2                      -.70310               .60425          -1.1636[.278]
 dLNK2                      -3.1199               2.2767          -1.3704[.208]
 dLNR2                      .065539               .14161           .46282[.656]
 dLNY3                      -.21946               .30151          -.72787[.487]
 dLNK3                      -3.8471               2.5847          -1.4884[.175]
 dLNR3                     -.037545               .10701          -.35087[.735]
 ecm1(-1)                    .98145               .64354           1.5251[.166]
 ecm2(-1)                   -.10690               .18290          -.58445[.575]
******************************************************************************
 List of additional temporary variables created:
 dLNY = LNY-LNY(-1)
 dLNY1 = LNY(-1)-LNY(-2)
 dLNK1 = LNK(-1)-LNK(-2)
 dLNR1 = LNR(-1)-LNR(-2)
 dLNY2 = LNY(-2)-LNY(-3)
 dLNK2 = LNK(-2)-LNK(-3)
 dLNR2 = LNR(-2)-LNR(-3)
 dLNY3 = LNY(-3)-LNY(-4)
 dLNK3 = LNK(-3)-LNK(-4)
 dLNR3 = LNR(-3)-LNR(-4)
 ecm1 =    1.0000*LNY   -1.4336*LNK      0.00*LNR  -.022666*Trend;ecm2 =   -3.
3146*LNY      0.00*LNK +   1.0000*LNR +  .072030*Trend
******************************************************************************
 R-Squared                    .63698   R-Bar-Squared                    .13782
 S.E. of Regression           .036993   F-stat.    F( 11,   8)    1.2761[.373]
 Mean of Dependent Variable  .0076881   S.D. of Dependent Variable     .039840
 Residual Sum of Squares      .010948   Equation Log-likelihood        46.7246
 Akaike Info. Criterion       34.7246   Schwarz Bayesian Criterion     28.7502
 DW-statistic                  1.9986   System Log-likelihood         178.6299
******************************************************************************


                               Diagnostic Tests
******************************************************************************
*    Test Statistics  *        LM Version        *         F Version         *
******************************************************************************
*                     *                          *                           *
* A:Serial Correlation*CHSQ(   1)=  .020188[.887]*F(   1,   7)= .0070729[.935]*
*                     *                          *                           *
* B:Functional Form   *CHSQ(   1)=   1.8872[.170]*F(   1,   7)=   .72934[.421]*
*                     *                          *                           *
* C:Normality         *CHSQ(   2)=  .015115[.992]*       Not applicable      *
*                     *                          *                           *
* D:Heteroscedasticity*CHSQ(   1)=   .61524[.433]*F(   1,  18)=   .57129[.460]*
******************************************************************************
   A:Lagrange multiplier test of residual serial correlation
   B:Ramsey's RESET test using the square of the fitted values
   C:Based on a test of skewness and kurtosis of residuals
```

- And:

```
     ECM for variable LNR estimated by OLS based on cointegrating VAR(4)
*******************************************************************************
 Dependent variable is dLNR
 20 observations used for estimation from 1977 to 1996
*******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 Intercept                   1.5627              16.0909            .097114[.925]
 dLNY1                       3.9892               4.0730             .97942[.356]
 dLNK1                      -2.8697              11.1846            -.25657[.804]
 dLNR1                      -.50032               .71475            -.70000[.504]
 dLNY2                       4.7931               2.4852             1.9286[.090]
 dLNK2                      -5.2192               9.3639            -.55737[.593]
 dLNR2                      -.45113               .58243            -.77457[.461]
 dLNY3                      -.52381               1.2401            -.42240[.684]
 dLNK3                      25.6764              10.6309             2.4153[.042]
 dLNR3                       .18629               .44010             .42329[.683]
 ecm1(-1)                   -2.5124               2.6468            -.94922[.370]
 ecm2(-1)                    .13686               .75226             .18194[.860]
*******************************************************************************
 List of additional temporary variables created:
 dLNR = LNR-LNR(-1)
 dLNY1 = LNY(-1)-LNY(-2)
 dLNK1 = LNK(-1)-LNK(-2)
 dLNR1 = LNR(-1)-LNR(-2)
 dLNY2 = LNY(-2)-LNY(-3)
 dLNK2 = LNK(-2)-LNK(-3)
 dLNR2 = LNR(-2)-LNR(-3)
 dLNY3 = LNY(-3)-LNY(-4)
 dLNK3 = LNK(-3)-LNK(-4)
 dLNR3 = LNR(-3)-LNR(-4)
 ecm1 =    1.0000*LNY   -1.4336*LNK      0.00*LNR  -.022666*Trend;ecm2 =   -3.
3146*LNY      0.00*LNK +   1.0000*LNR +  .072030*Trend
*******************************************************************************
 R-Squared                     .66447   R-Bar-Squared                   .20311
 S.E. of Regression            .15215   F-stat.    F( 11,   8)    1.4402[.309]
 Mean of Dependent Variable  -.034657   S.D. of Dependent Variable      .17044
 Residual Sum of Squares       .18520   Equation Log-likelihood        18.4419
 Akaike Info. Criterion        6.4419   Schwarz Bayesian Criterion      .46755
 DW-statistic                  2.0858   System Log-likelihood         178.6299
*******************************************************************************


                               Diagnostic Tests
*******************************************************************************
*    Test Statistics  *        LM Version        *         F Version          *
*******************************************************************************
*                     *                          *                            *
* A:Serial Correlation*CHSQ(   1)=   .80882[.368]*F(   1,   7)=   .29502[.604]*
*                     *                          *                            *
* B:Functional Form   *CHSQ(   1)=   3.4994[.061]*F(   1,   7)=   1.4845[.263]*
*                     *                          *                            *
* C:Normality         *CHSQ(   2)=   2.2545[.324]*       Not applicable       *
*                     *                          *                            *
* D:Heteroscedasticity*CHSQ(   1)=   .15419[.695]*F(   1,  18)=   .13985[.713]*
*******************************************************************************
   A:Lagrange multiplier test of residual serial correlation
   B:Ramsey's RESET test using the square of the fitted values
   C:Based on a test of skewness and kurtosis of residuals
```

• And Persistence Profile of Shock to CV's:



5. Example: Inflation in South Africa

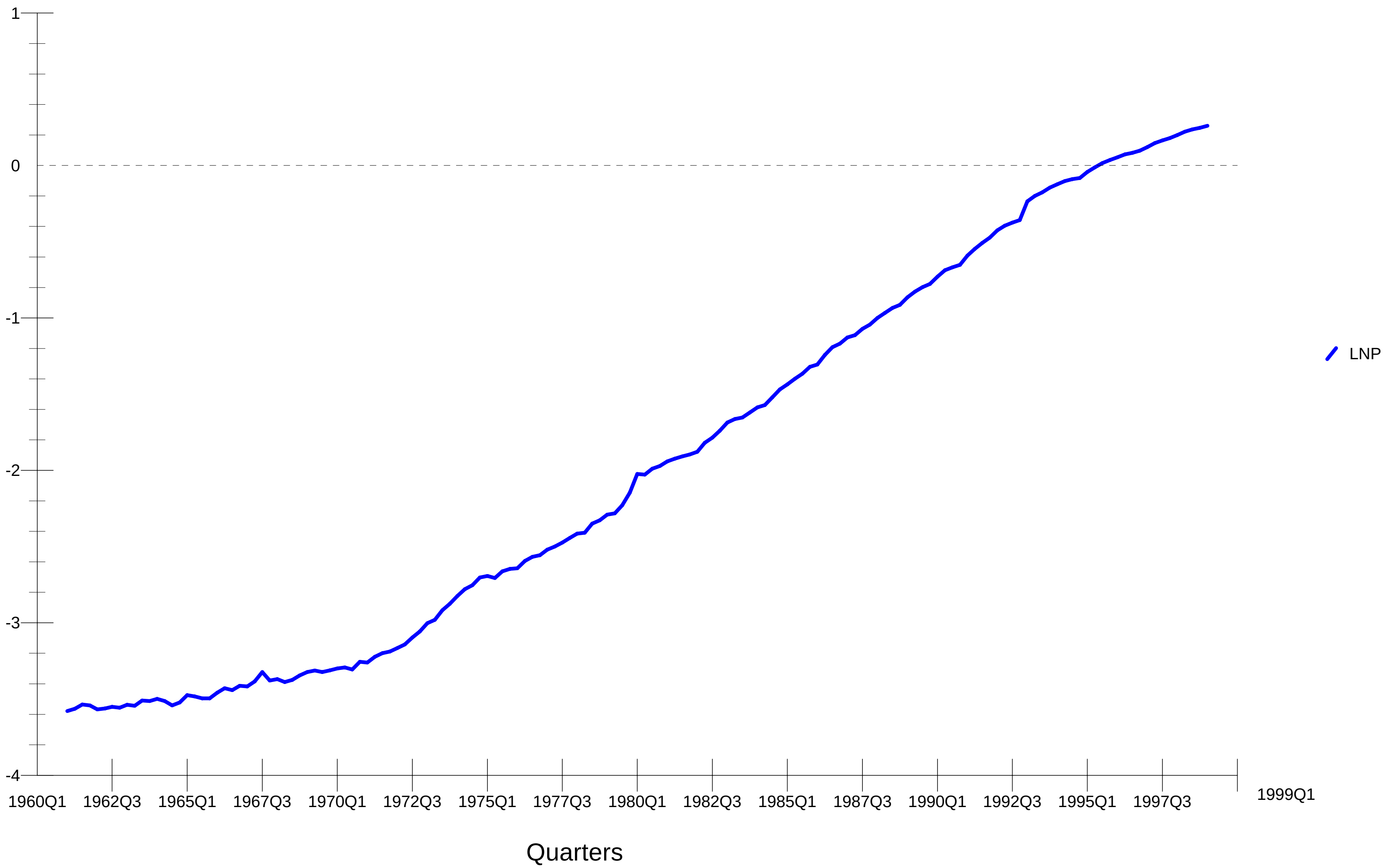
- Fedderke & Schaling (2005):
- The model is an expectations augmented Phillips curve model:

$$p_t = \alpha_0 + \alpha_1 (w_t - q_t) + \alpha_2 \widehat{y}_t + \alpha_3 S_t + \varepsilon_{1t} \quad (21)$$

$$(w_t - q_t) = \beta_0 + \beta_1 p_t^e + \beta_2 \widehat{y}_t + \beta_3 S_t + \varepsilon_{2t} \quad (22)$$

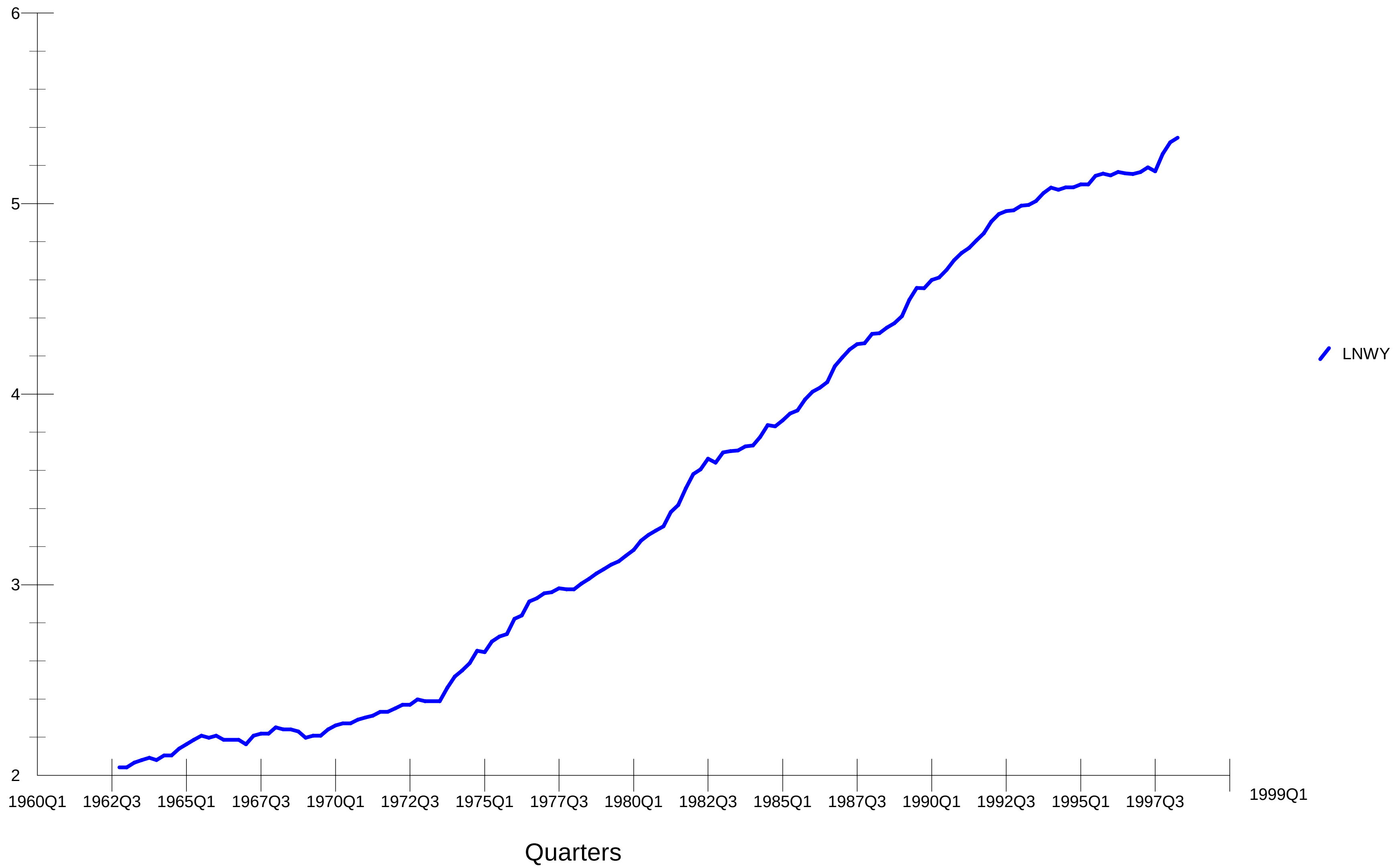
where p_t denotes the price level (given by the GDP deflator), $(w_t - q_t)$ unit labour cost with w_t denoting nominal wage rates and q_t labour productivity, $\widehat{y}_t$ denotes the output gap as an indication of demand shocks, S_t denotes the real exchange rate (as proxy for supply shocks), and p_t^e price expectations.

- We have:



1
0
-1
-2
-3
-4
1960Q1
1962Q3
1965Q1
1967Q3
1970Q1
1972Q3
1975Q1
1977Q3
1980Q1
1982Q3
1985Q1
1987Q3
1990Q1
1992Q3
1995Q1
1997Q3
1999Q1
Quarters
LNP

• and:

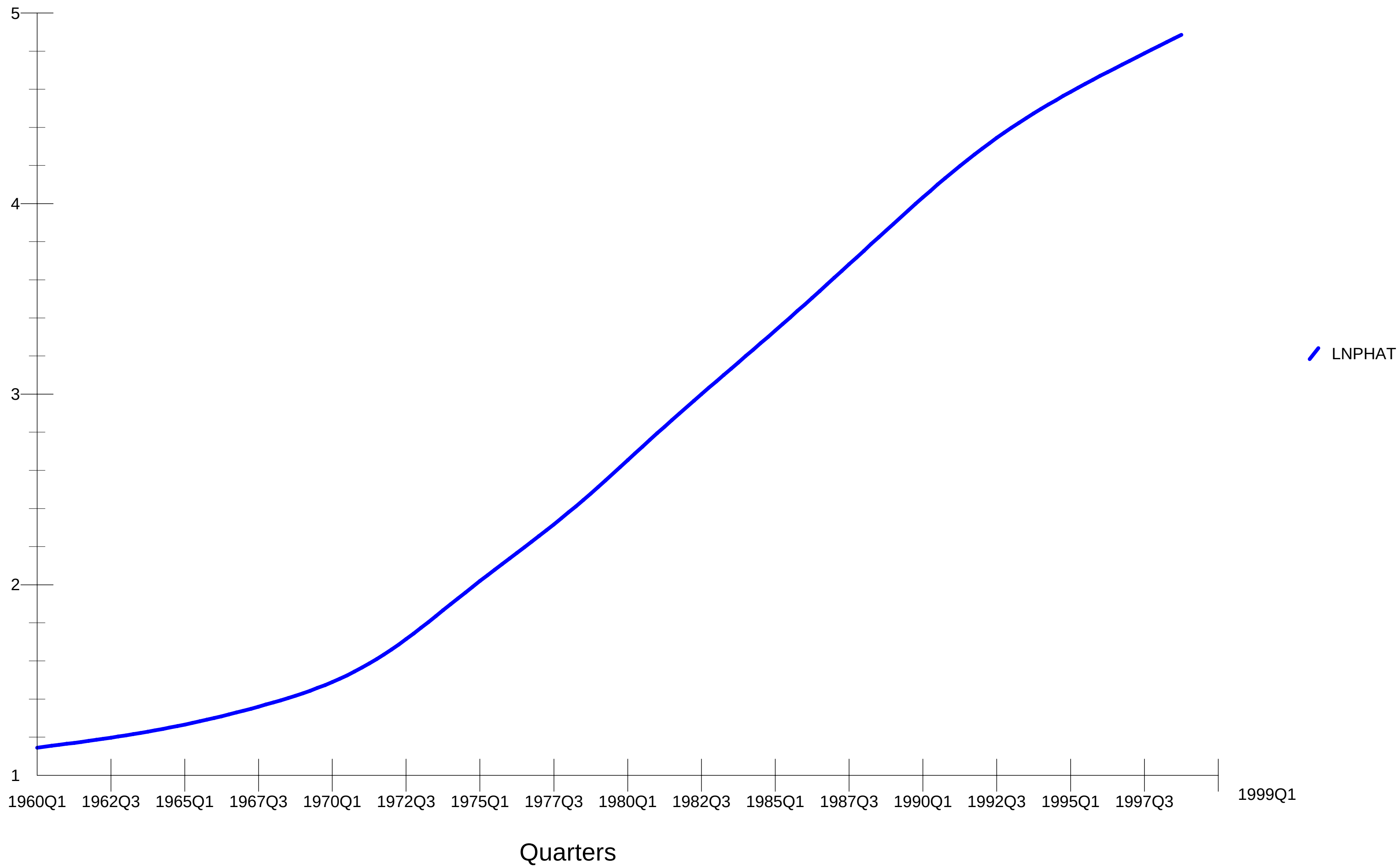


6
5
4
3
2
LNWY
1960Q1
1962Q3
1965Q1
1967Q3
1970Q1
1972Q3
1975Q1
1977Q3
1980Q1
1982Q3
1985Q1
1987Q3
1990Q1
1992Q3
1995Q1
1997Q3
1999Q1
Quarters

- as well as:

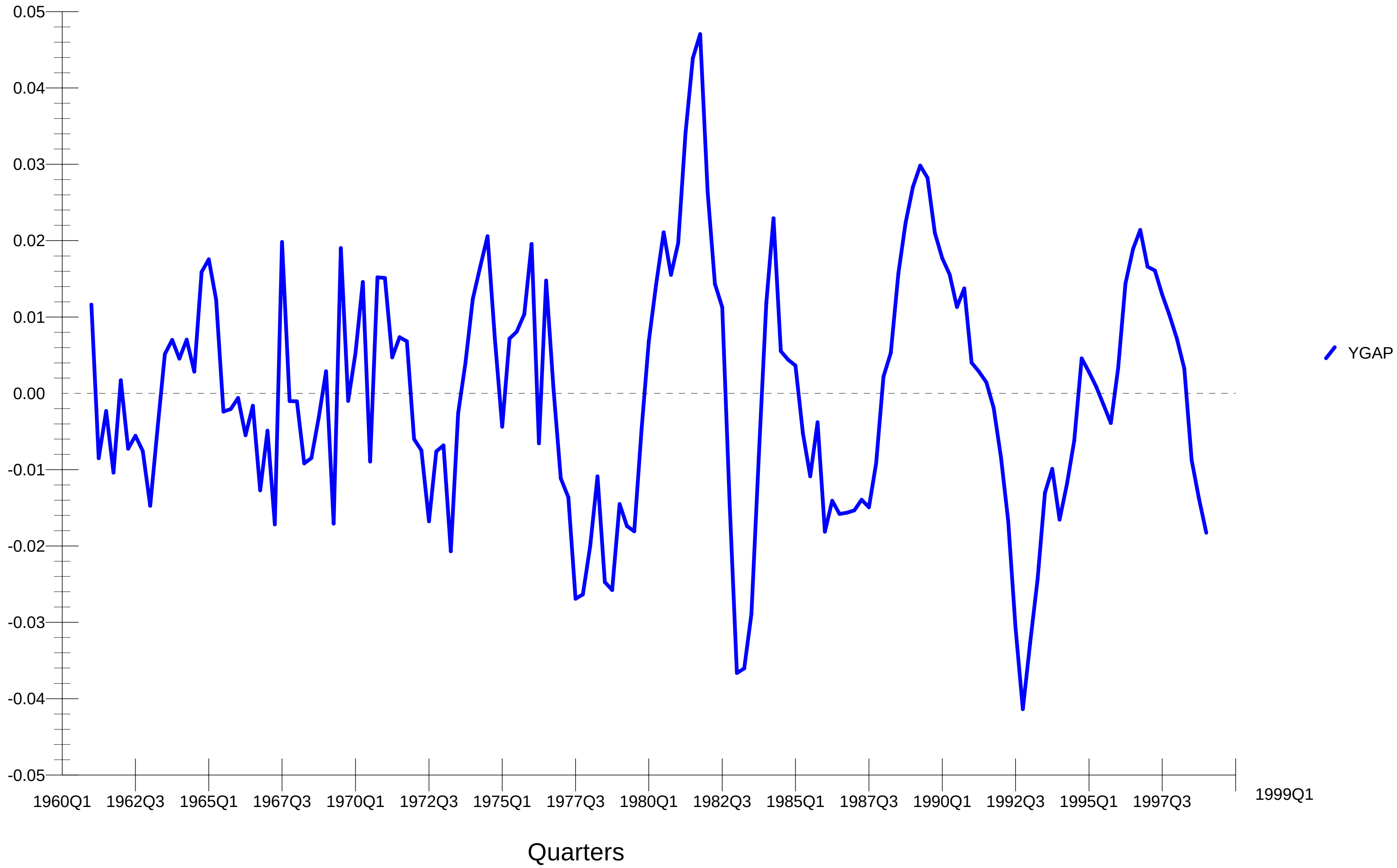
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
1960Q1
1962Q3
1965Q1
1967Q3
1970Q1
1972Q3
1975Q1
1977Q3
1980Q1
1982Q3
1985Q1
1987Q3
1990Q1
1992Q3
1995Q1
1997Q3
1999Q1
Quarters
LNREX

- and then:



5
4
3
2
1
1960Q1
1962Q3
1965Q1
1967Q3
1970Q1
1972Q3
1975Q1
1977Q3
1980Q1
1982Q3
1985Q1
1987Q3
1990Q1
1992Q3
1995Q1
1997Q3
1999Q1
Quarters
LNPHAT

• and finally:



0.05
0.04
0.03
0.02
0.01
0.00
-0.01
-0.02
-0.03
-0.04
-0.05
1960Q1
1962Q3
1965Q1
1967Q3
1970Q1
1972Q3
1975Q1
1977Q3
1980Q1
1982Q3
1985Q1
1987Q3
1990Q1
1992Q3
1995Q1
1997Q3
1999Q1
Quarters
YGAP

- In this case we expect $r = 2$, and for the long run parameters:

$$\Pi z_{t-k+1} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \\ \alpha_{41} & \alpha_{42} \end{bmatrix} \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ \beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} \end{bmatrix} \begin{bmatrix} p \\ w - q \\ p^e \\ S \end{bmatrix}_{t-k+1} \tag{23}$$

- Cointegrating relationships are provided by $\varepsilon_i = \beta_{i1}p + \beta_{i2}(w - q) + \beta_{i3}p^e + \beta_{i4}S$, with the α_{ij} providing the error correction terms.
- Exact identification requires r^2 restrictions, for the expectation that $r = 2$ thus 4.
- On the basis of the theoretical discussion we specify:

$$\Pi z_{t-k+1} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \\ \alpha_{41} & \alpha_{42} \end{bmatrix} \begin{bmatrix} 1 & -\beta_{12} & 0 & \beta_{14} \\ 0 & 1 & -\beta_{23} & \beta_{24} \end{bmatrix} \begin{bmatrix} p \\ w - q \\ p^e \\ S \end{bmatrix}_{t-k+1}$$

- The univariate time series characteristics of the data are given by:

	$\sim I(0)$	$\sim I(1)$
p_t	.50	-3.15*
$(w_t - q_t)$	1.23	-4.46*
$\widehat{y}_t$	-5.34*	
S_t	-2.17	-8.63*
p_t^e	.58	-5.44*

- max-λ:

```
     Cointegration with unrestricted intercepts and no trends in the VAR
   Cointegration LR Test Based on Maximal Eigenvalue of the Stochastic Matrix
*******************************************************************************
 139 observations from 1963Q4 to 1998Q2. Order of VAR = 4.
 List of variables included in the cointegrating vector:
 LNP             LNWY            LNREX           LNPHAT
 List of I(0) variables included in the VAR:
 YGAP            DU1             DU2
 List of eigenvalues in descending order:
.53748     .17403    .076279   .0080622
*******************************************************************************
 Null    Alternative    Statistic     95% Critical Value     90%Critical Value
 r = 0      r = 1       107.1776           27.4200                24.9900
 r<= 1      r = 2        26.5765           21.1200                19.0200
 r<= 2      r = 3        11.0290           14.8800                12.9800
 r<= 3      r = 4         1.1252            8.0700                 6.5000
*******************************************************************************
 Use the above table to determine r (the number of cointegratingvectors).
```

- trace:

```
     Cointegration with unrestricted intercepts and no trends in the VAR
         Cointegration LR Test Based on Trace of the Stochastic Matrix
*******************************************************************************
 139 observations from 1963Q4 to 1998Q2. Order of VAR = 4.
 List of variables included in the cointegrating vector:
 LNP            LNWY           LNREX          LNPHAT
 List of I(0) variables included in the VAR:
 YGAP           DU1            DU2
 List of eigenvalues in descending order:
.53748     .17403    .076279   .0080622
*******************************************************************************
 Null    Alternative    Statistic     95% Critical Value     90%Critical Value
 r = 0      r>= 1       145.9083           48.8800                45.7000
 r<= 1      r>= 2        38.7308           31.5400                28.7800
 r<= 2      r>= 3        12.1542           17.8600                15.7500
 r<= 3      r = 4         1.1252            8.0700                 6.5000
*******************************************************************************
 Use the above table to determine r (the number of cointegratingvectors).
```

$$\Longrightarrow r = 2$$

- Thus:

```
      ML estimates subject to exactly identifying restriction(s)
    Estimates of Restricted Cointegrating Relations (SE's in Brackets)
                   Converged after 2 iterations
    Cointegration with unrestricted intercepts and no trends in the VAR
*******************************************************************************
 139 observations from 1963Q4 to 1998Q2. Order of VAR = 4, chosen r =2.
 List of variables included in the cointegrating vector:
 LNP             LNWY            LNREX           LNPHAT
 List of I(0) variables included in the VAR:
 YGAP            DU1             DU2
*******************************************************************************
 List of imposed restriction(s) on cointegrating vectors:
 A1=1; A4=0;  B1=0;B2=1;
*******************************************************************************
                  Vector  1      Vector  2
 LNP                  1.0000           0.00
                  (   *NONE*)    (   *NONE*)

 LNWY                -1.3101         1.0000
                  (   .11627)    (   *NONE*)

 LNREX               -.21573       -.026146
                  (   .35688)    (   .17944)

 LNPHAT                 0.00        -.86145
                  (   *NONE*)    (  .050008)

*******************************************************************************
 LL subject to exactly identifying restrictions=   2372.7
*******************************************************************************
```

- And:

```
     ECM for variable LNP estimated by OLS based on cointegrating VAR(4)
******************************************************************************
 Dependent variable is dLNP
 139 observations used for estimation from 1963Q4 to 1998Q2
******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 Intercept                 -.90291              .25277           -3.5721[.001]
 dLNP1                    -.034629              .11477           -.30172[.763]
 dLNWY1                    .085690             .084488            1.0142[.312]
 dLNREX1                  -.018617             .027642           -.67351[.502]
 dLNPHAT1                -132.8989             40.3735           -3.2917[.001]
 dLNP2                    -.056453              .10893           -.51823[.605]
 dLNWY2                    -.10409             .082921           -1.2552[.212]
 dLNREX2                  .7100E-3             .027793           .025546[.980]
 dLNPHAT2                 237.5795             80.4768            2.9521[.004]
 dLNP3                     -.12620              .10882           -1.1597[.248]
 dLNWY3                    .016022             .084650            .18928[.850]
 dLNREX3                  -.020737             .026559           -.78077[.436]
 dLNPHAT3                -105.1069             40.9054           -2.5695[.011]
 ecm1(-1)                  -.21706             .061894           -3.5069[.001]
 ecm2(-1)                  -.46059              .14544           -3.1669[.002]
 YGAP                      -.31084              .11629           -2.6730[.009]
 DU1                       .015509             .012429            1.2479[.214]
 DU2                      -.010539             .010583           -.99580[.321]
******************************************************************************

******************************************************************************
 R-Squared                      .41489   R-Bar-Squared                   .33269
 S.E. of Regression            .019238   F-stat.    F( 17, 121)    5.0471[.000]
 Mean of Dependent Variable    .026851   S.D. of Dependent Variable     .023550
 Residual Sum of Squares       .044782   Equation Log-likelihood       361.5775
 Akaike Info. Criterion       343.5775   Schwarz Bayesian Criterion    317.1672
 DW-statistic                   2.0942   System Log-likelihood           2372.7
******************************************************************************


                                Diagnostic Tests
******************************************************************************
*    Test Statistics  *        LM Version       *         F Version          *
******************************************************************************
*                     *                         *                            *
* A:Serial Correlation*CHSQ(   4)=   9.3063[.054]*F(   4, 117)=   2.0989[.085]*
*                     *                         *                            *
* B:Functional Form   *CHSQ(   1)= .0092445[.923]*F(   1, 120)= .0079814[.929]*
*                     *                         *                            *
* C:Normality         *CHSQ(   2)=  89.4983[.000]*       Not applicable      *
*                     *                         *                            *
* D:Heteroscedasticity*CHSQ(   1)=   1.7746[.183]*F(   1, 137)=   1.7717[.185]*
******************************************************************************
   A:Lagrange multiplier test of residual serial correlation
   B:Ramsey's RESET test using the square of the fitted values
   C:Based on a test of skewness and kurtosis of residuals
```

- As well as:

ECM for variable LNWY estimated by OLS based on cointegrating VAR(4)

Dependent variable is dLNWY
139 observations used for estimation from 1963Q4 to 1998Q2

Regressor	Coefficient	Standard Error	T-Ratio[Prob]
Intercept	-1.0223	.26873	-3.8041[.000]
dLNP1	.13603	.12202	1.1148[.267]
dLNWY1	.10320	.089823	1.1489[.253]
dLNREX1	-.057232	.029387	-1.9475[.054]
dLNPHAT1	36.6659	42.9231	.85422[.395]
dLNP2	.24519	.11581	2.1171[.036]
dLNWY2	.26218	.088157	2.9740[.004]
dLNREX2	-.041652	.029548	-1.4096[.161]
dLNPHAT2	-124.8648	85.5590	-1.4594[.147]
dLNP3	.23049	.11569	1.9922[.049]
dLNWY3	.16552	.089996	1.8391[.068]
dLNREX3	-.9403E-3	.028236	-.033300[.973]
dLNPHAT3	86.4756	43.4886	1.9885[.049]
ecm1(-1)	-.27960	.065802	-4.2492[.000]
ecm2(-1)	-.73702	.15462	-4.7666[.000]
YGAP	.34854	.12363	2.8191[.006]
DU1	.0054820	.013213	.41489[.679]
DU2	-.0045816	.011251	-.40720[.685]

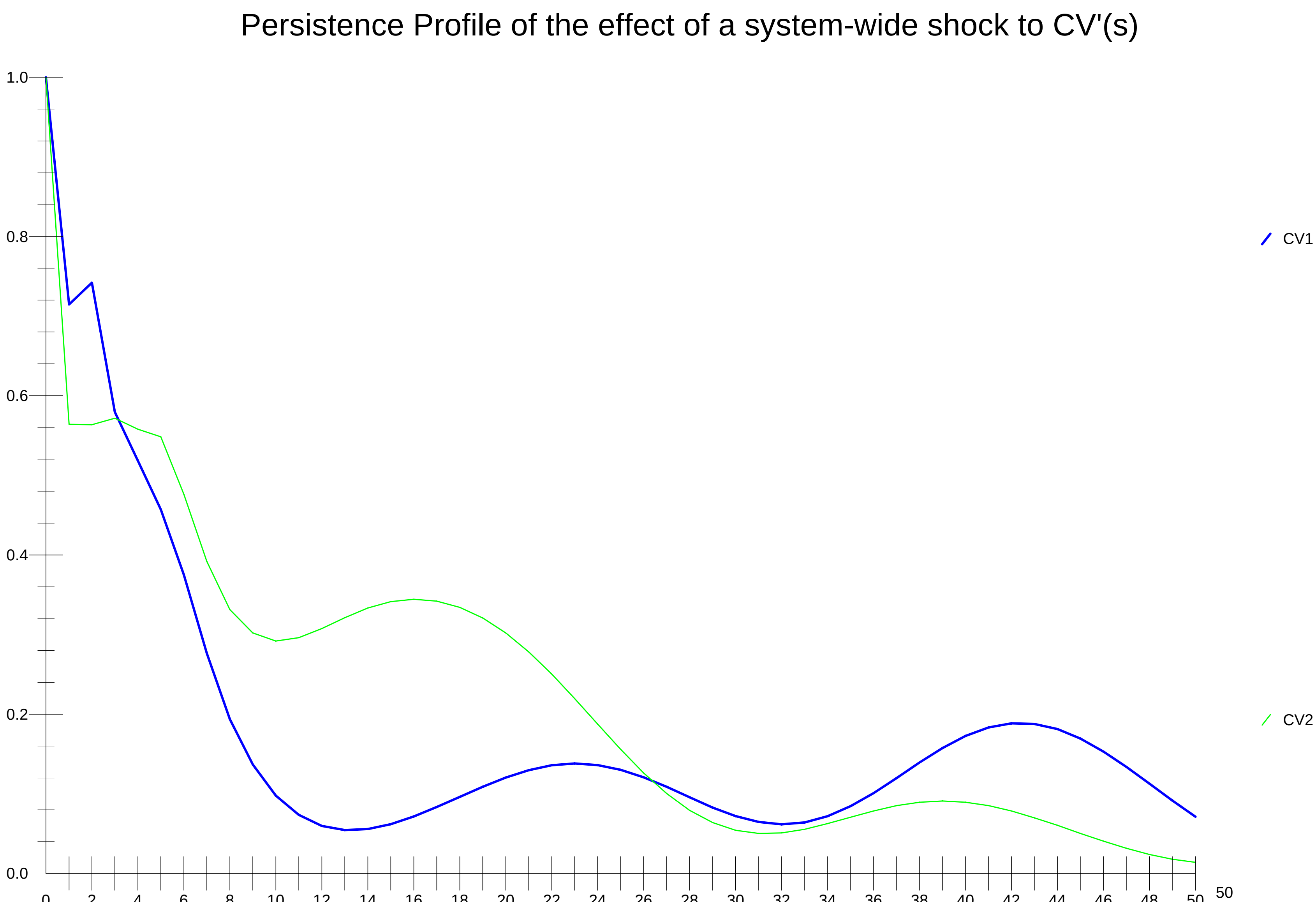
R-Squared	.42151	R-Bar-Squared	.34023
S.E. of Regression	.020453	F-stat. F(17, 121)	5.1861[.000]
Mean of Dependent Variable	.023502	S.D. of Dependent Variable	.025180
Residual Sum of Squares	.050616	Equation Log-likelihood	353.0656
Akaike Info. Criterion	335.0656	Schwarz Bayesian Criterion	308.6553
DW-statistic	1.9970	System Log-likelihood	2372.7

Diagnostic Tests

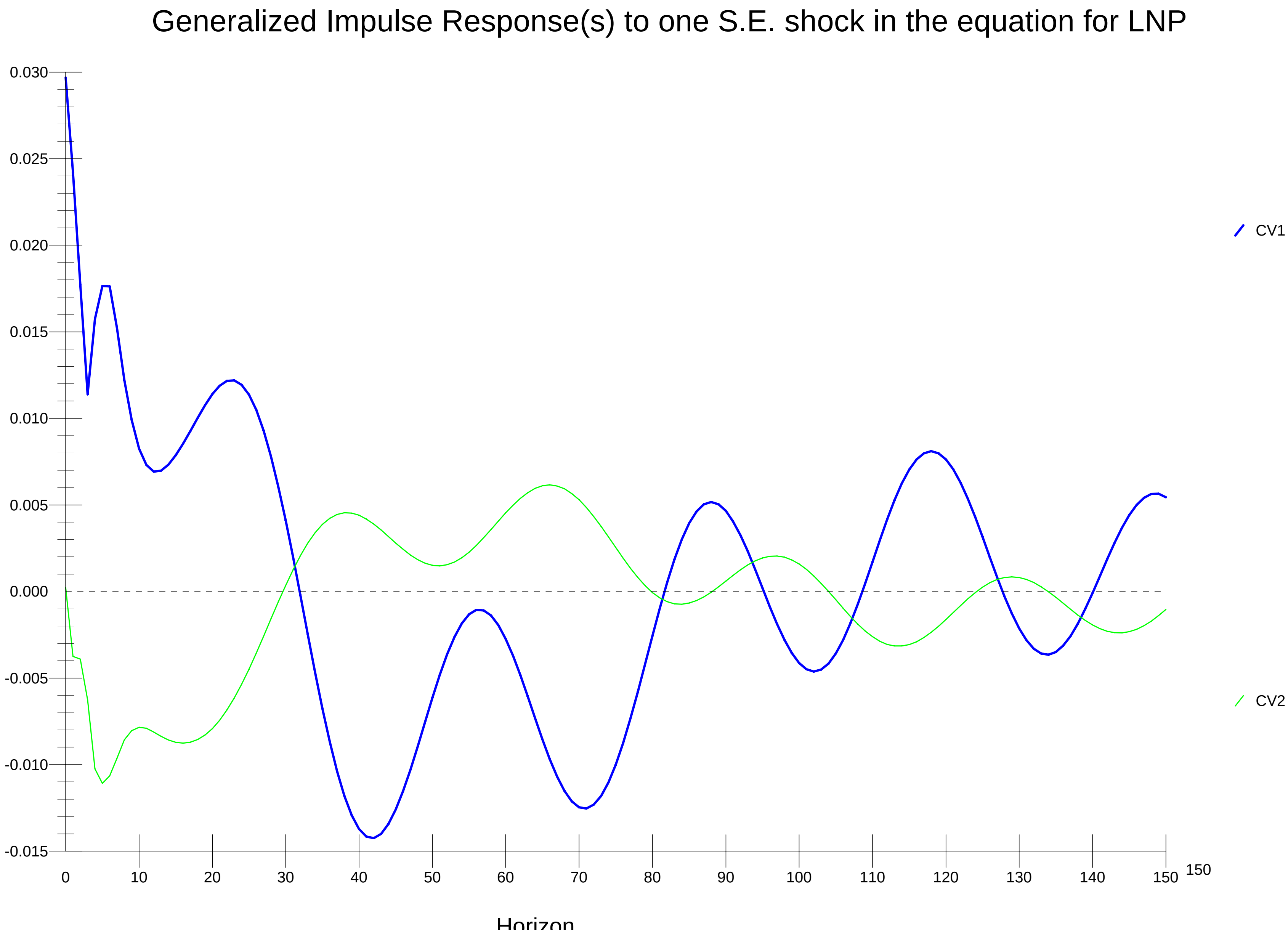
Test Statistics	LM Version	F Version
A:Serial Correlation	CHSQ(4)= 3.5268[.474]	F(4, 117)= .76146[.552]
B:Functional Form	CHSQ(1)= 3.9090[.048]	F(1, 120)= 3.4723[.065]
C:Normality	CHSQ(2)= .0058665[.997]	Not applicable
D:Heteroscedasticity	CHSQ(1)= 4.1867[.041]	F(1, 137)= 4.2546[.041]

A:Lagrange multiplier test of residual serial correlation
B:Ramsey's RESET test using the square of the fitted values
C:Based on a test of skewness and kurtosis of residuals
D:Based on the regression of squared residuals on squared fitted values

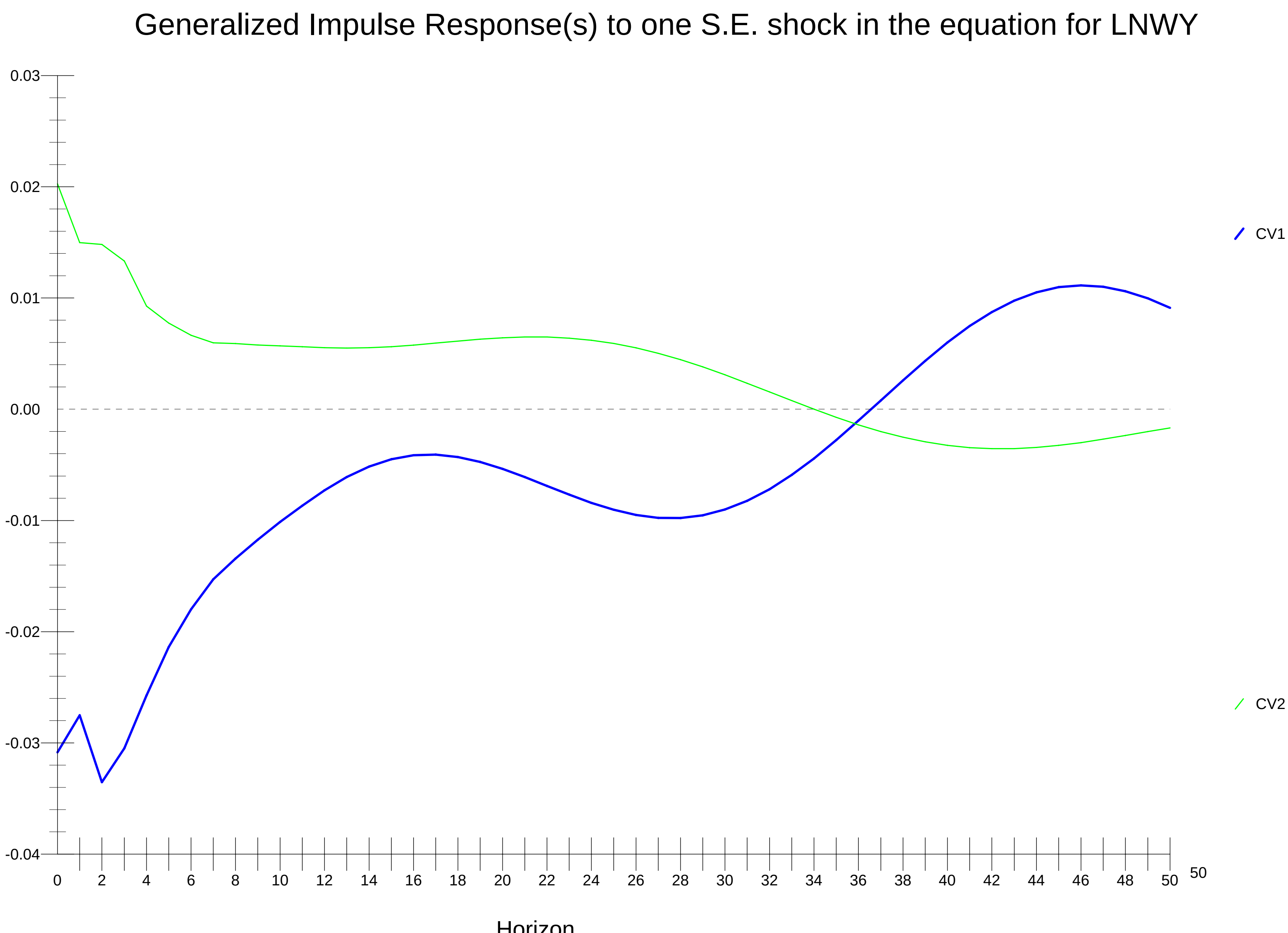
- With Persistence Profile of System-wide Shocks to CV's:



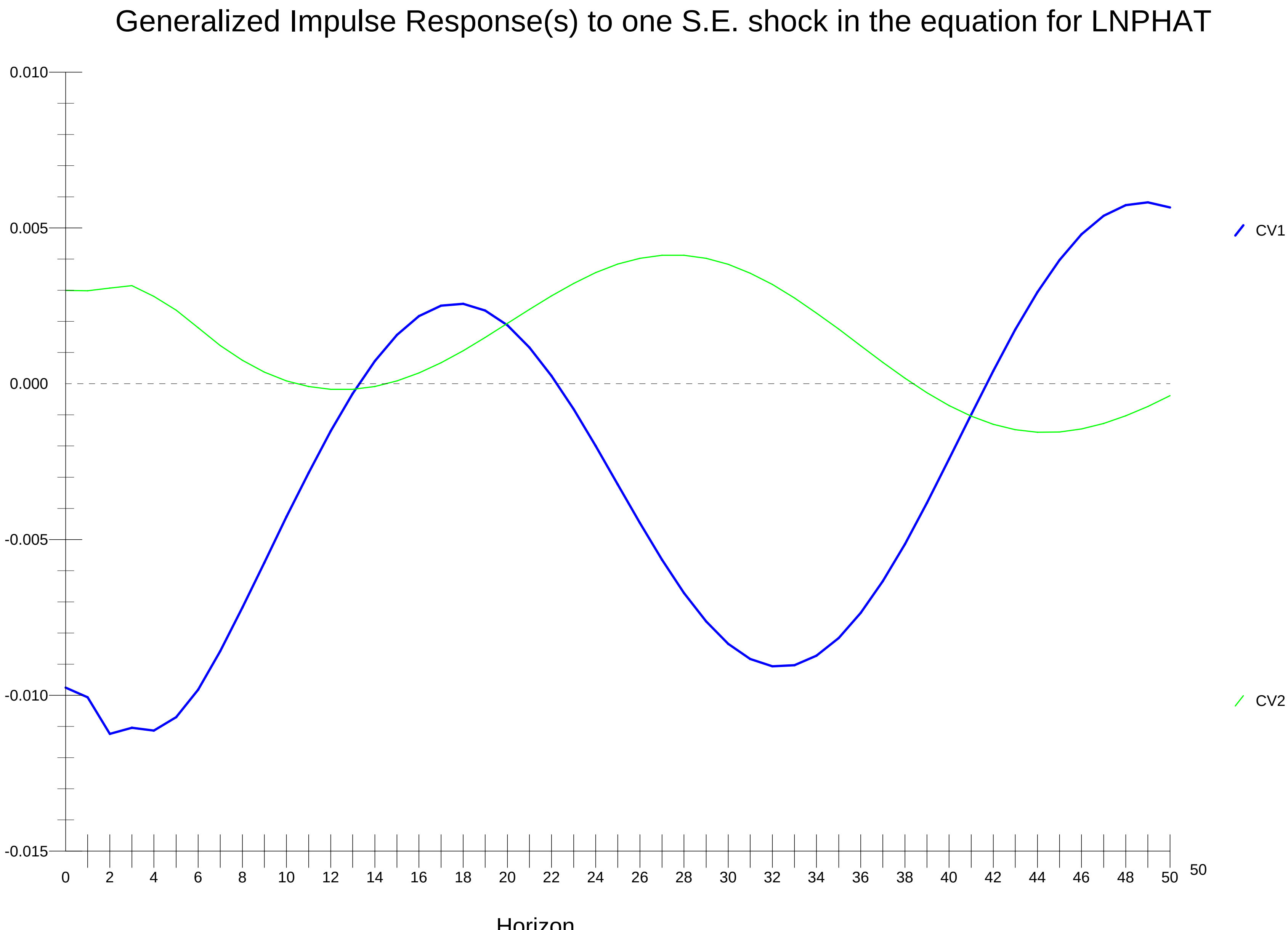
- Impulse response to lnP to CV's:



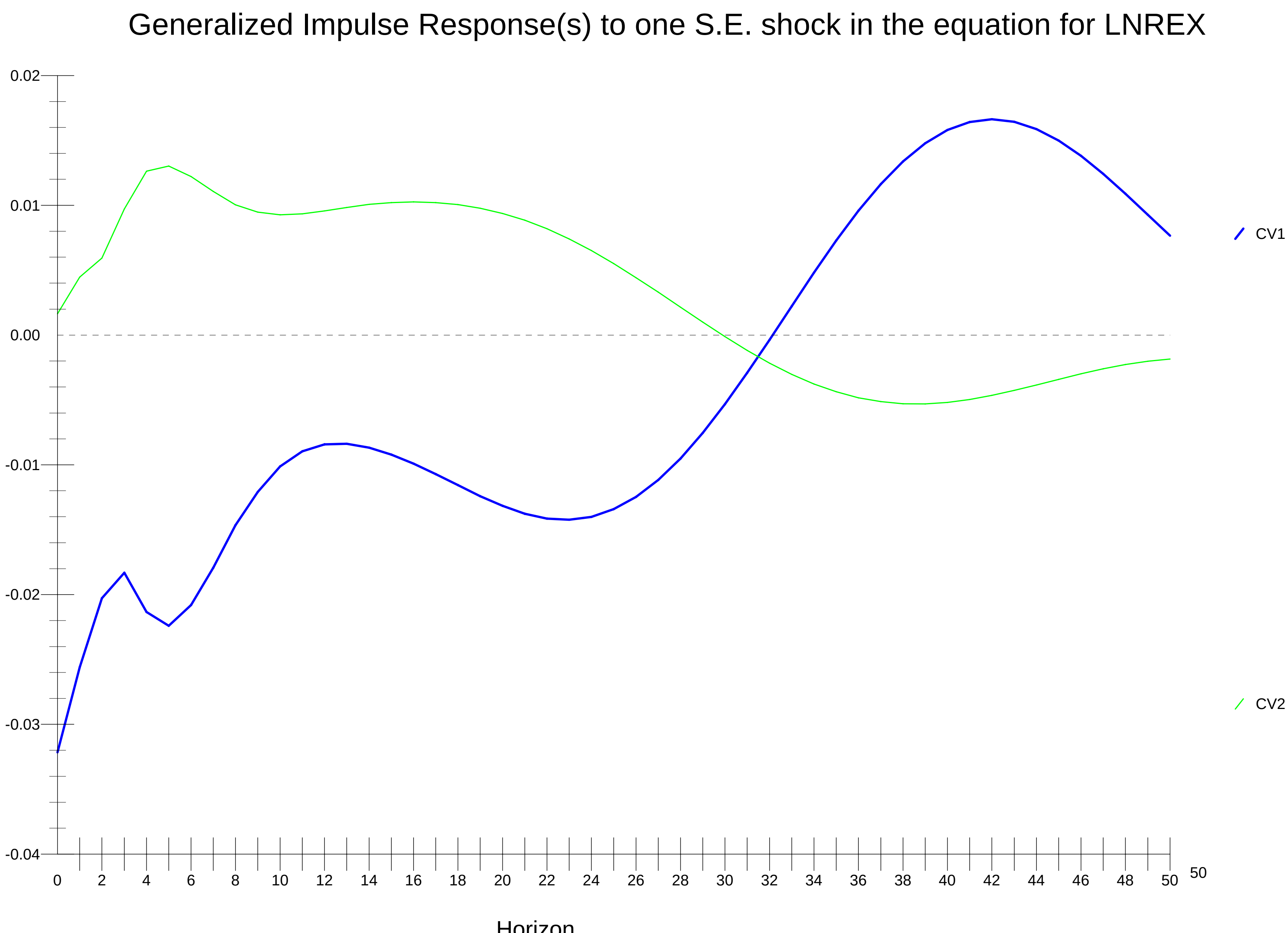
- Impulse response to lnWY to CV's:



- Impulse response to lnPHAT to CV's:



- Impulse response to lnREX to CV's:



- Impulse response of lnP, lnWY, lnPHAT, lnREX to shock in lnP equation: