Applied Time Series Econometrics

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Establishing the Univariate Characteristics of the Data

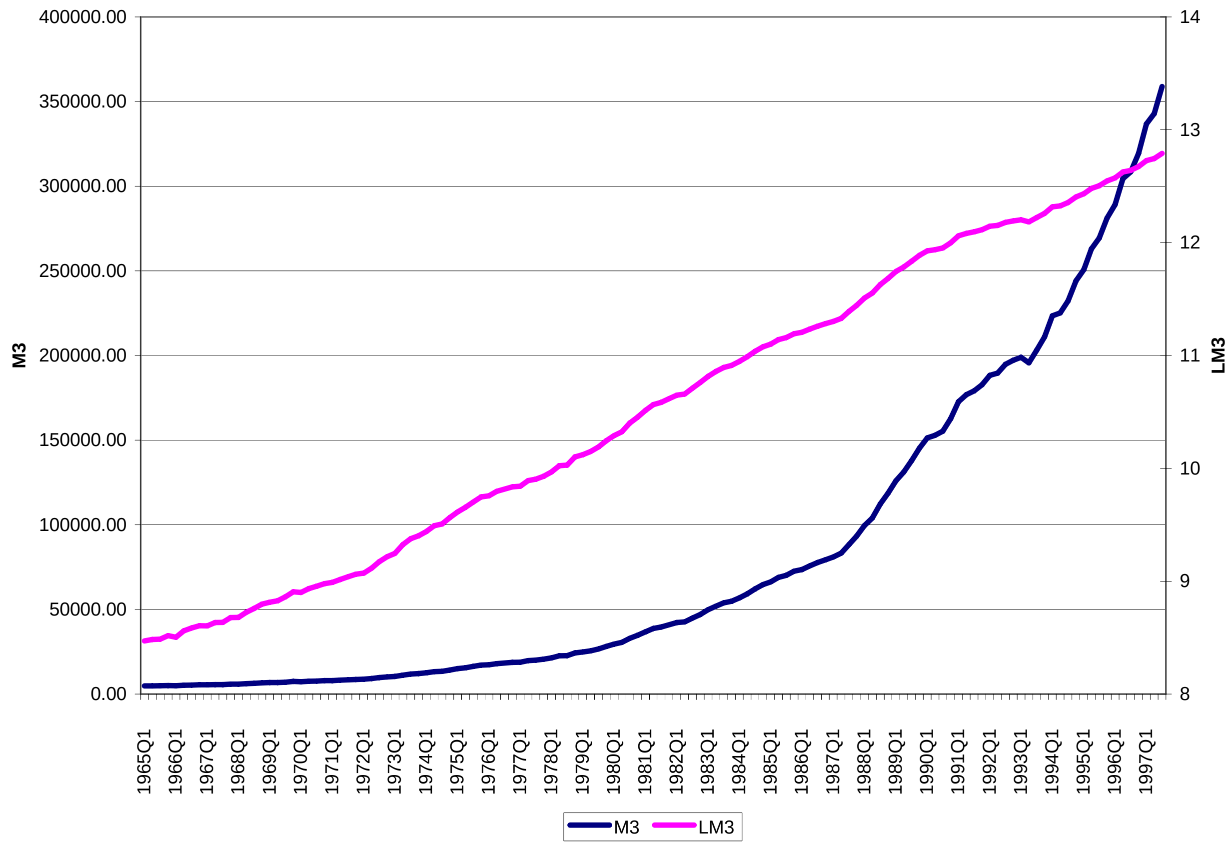
Have demonstrated the importance of stationarity of data to estimation.

Now:

- *Formal* tests which examine the stationarity properties of the data to be used.
- Rest on an examination of residuals of DGP's.
- Data transformations.

1. Establishing the Appropriate Variable Transformations

Consider:



- Note: data series *should* be integrated in only one of the two scales.
- Illustration: macroeconomic series often grow over time subject to some stochastic shocks. Hence:

$$\begin{aligned} Y_t &= (1+\gamma) Y_{t-1} e_t, \\ e_t &\sim ILN\left(1, \sigma^2\right) \\ \gamma &= constant \end{aligned}$$

constant.

- It follows:

$$\begin{aligned} \ln Y_t &= \ln(1+\gamma) + \ln Y_{t-1} + \ln e_t \\ or \quad y_t &\approx \gamma + y_{t-1} + \epsilon_t \quad \epsilon_t \sim IN\left(0, \sigma^2\right) \\ \therefore \quad \Delta y_t &= \gamma + \epsilon_t \end{aligned}$$

$\Longrightarrow$ variable grows at a constant proportional rate of γ, subject to random shocks.

$\Longrightarrow$ For (most) macroeconomic series growth rate varies about a (typically constant) mean,

$\Longrightarrow y_t \sim I(1)$, $\Delta y_t \sim I(0)$.

- But then it is not possible for $Y_t \sim I(1)$, since:

$$\begin{aligned} Y_t &= \alpha_0 + \alpha_1 Y_{t-1} + u_t, \quad \alpha_1 = 1 \\ \Longrightarrow \quad \Delta Y_t &= \alpha_0 + u_t \end{aligned}$$

  if $\Delta Y_t \sim I(0)$,
  $\Longrightarrow \Delta Y_t \sim \left(\alpha_0, \sigma^2\right)$
  $\Longrightarrow$ *declining* proportional growth rate in the series over time.

- It is not possible for both $\Delta y_t \sim I(0)$ and $\Delta Y_t \sim I(0)$ to be true.

- Are there formal tests available?
- Yes:
  - Ermini-Hendry
  - Schwarz Bayesian.

1.1 Ermini-Hendry Test

- Banerjee, Dolado, Galbraith and Hendry 1993:192f
- Estimate:

$$\Delta y_t = \mu + \varepsilon_t$$

  to formulate:

$$\lambda = \mu + (\sigma^2/2)$$

- Then estimate:

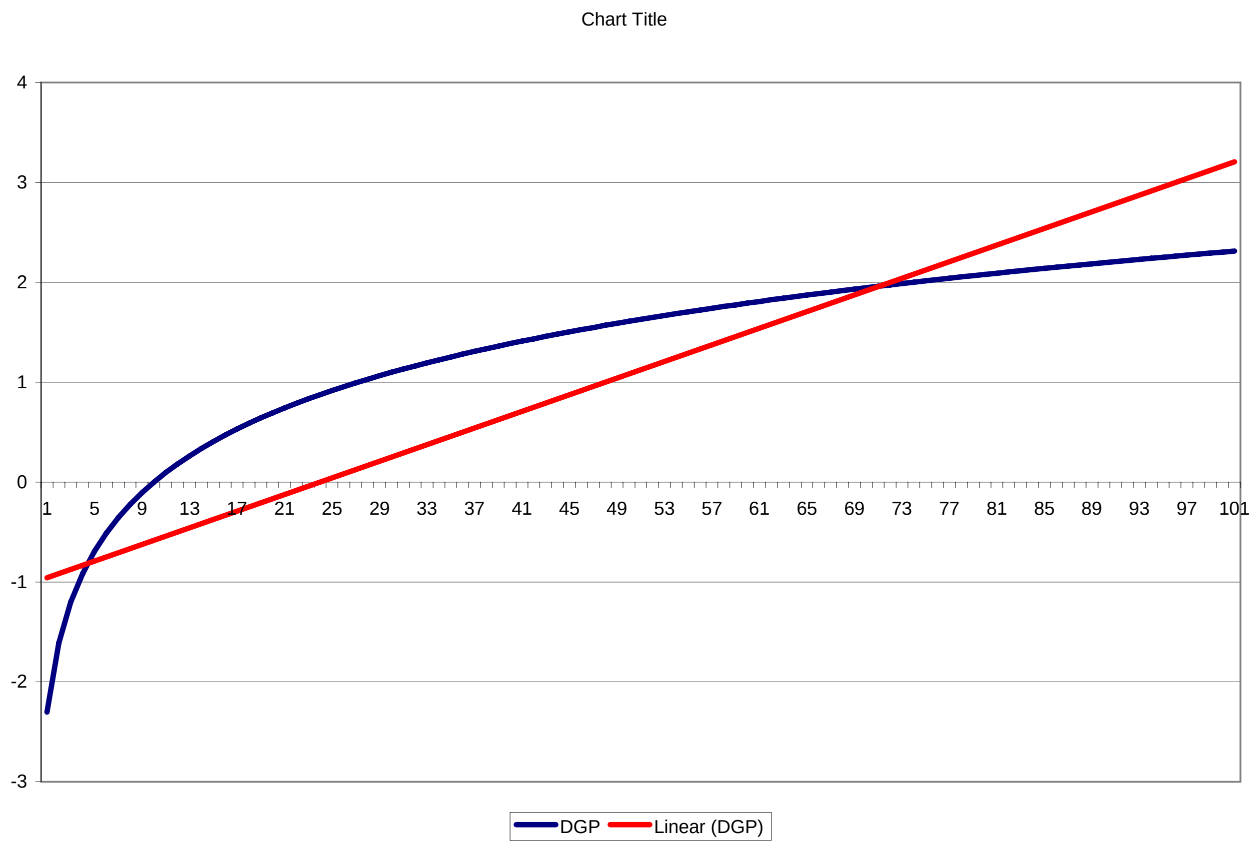
$$\Delta Y_t = \gamma + \sum \beta_i \Delta Y_{t-i} + \delta \exp(\lambda t)$$

- Lag length chosen to eliminate serial correlation.
- Test: log-transform encompasses model in levels if:

$$\gamma = 0, \delta \neq 0$$

- Intuition:
  - if the data are generated by the logarithmic model:
$$\Delta y_t = \mu + \varepsilon_t$$
  - model in levels would be characterized by drift:
$$\delta \exp(\lambda t)$$
    and variance:
$$\phi \exp[(2\lambda + \sigma^2)]$$
    exponentially increasing with time.
    $\Longrightarrow$ drift in model with normal scale *solely* due to underlying log-DGP.

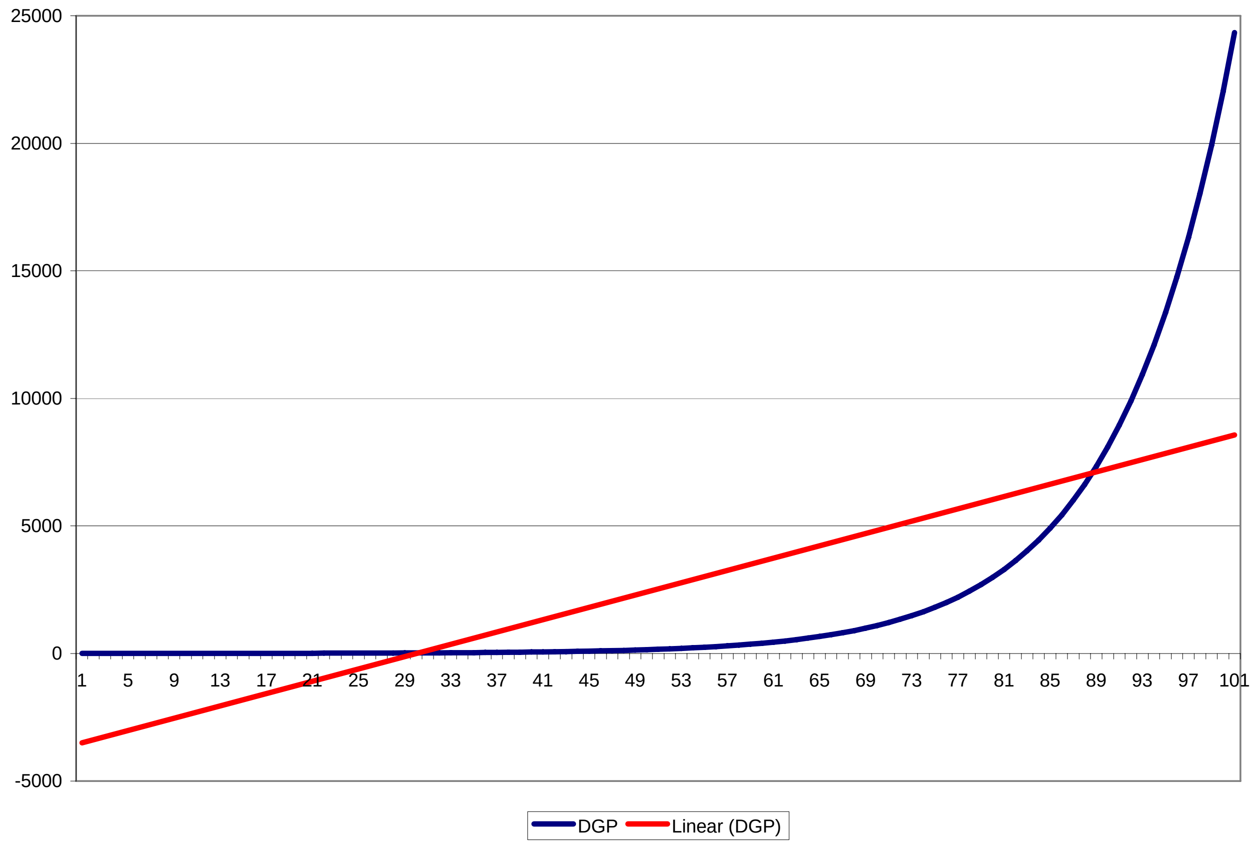
- For instance:



$$\Longrightarrow t \to \infty,\ \sigma^2 \to \infty$$

$$\Longrightarrow t \to \infty,\ \gamma < 0,\ \text{negative drift}$$

- Similarly:



25000
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101
DGP
Linear (DGP)

$$\Longrightarrow t \to \infty,\ \sigma^2 \to \infty$$

$$\Longrightarrow t \to \infty,\ \gamma > 0, \text{ positive drift}$$

- Example: South African M3
- Estimate:

$$\ln\left(\frac{M3}{P}\right)_t = \mu + \epsilon_t$$

- Gives:

```
                       Ordinary Least Squares Estimation
*******************************************************************************
 Dependent variable is DLM3
 130 observations used for estimation from 1965Q2 to 1997Q3
*******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 C                         .033239            .0016539            20.0968[.000]
*******************************************************************************
 R-Squared                        .0000   R-Bar-Squared                   .0000
 S.E. of Regression             .018858   F-stat.                        *NONE*
 Mean of Dependent Variable     .033239   S.D. of Dependent Variable    .018858
 Residual Sum of Squares        .045874   Equation Log-likelihood      332.2484
 Akaike Info. Criterion        331.2484   Schwarz Bayesian Criterion   329.8146
 DW-statistic                    1.7340
*******************************************************************************


                               Diagnostic Tests
*******************************************************************************
*    Test Statistics  *        LM Version        *         F Version          *
*******************************************************************************
*                     *                          *                            *
* A:Serial Correlation*CHSQ(   4)=  16.7788[.002]*F(   4, 125)=   4.6311[.002]*
*                     *                          *                            *
* B:Functional Form   *CHSQ(   1)=   *NONE*      *F(   1, 128)=   *NONE*      *
*                     *                          *                            *
* C:Normality         *CHSQ(   2)=   .77190[.680]*       Not applicable       *
*                     *                          *                            *
* D:Heteroscedasticity*CHSQ(   1)=   *NONE*      *F(   1, 128)=   *NONE*      *
*******************************************************************************
   A:Lagrange multiplier test of residual serial correlation
   B:Ramsey's RESET test using the square of the fitted values
   C:Based on a test of skewness and kurtosis of residuals
   D:Based on the regression of squared residuals on squared fitted values
```

- With:

$$\begin{aligned} \mu &= 0.033239 \\ \widehat{\sigma}^2 &= (0.018858)^2 \end{aligned}$$

- Hence:

$$\lambda = 0.033239 + \frac{(0.018858)^2}{2}$$

- And then estimate:

$$\Delta M3_t = \gamma + \sum \beta_i \Delta M3_{t-i} + \delta \exp(\lambda t) + \epsilon_t$$

- Gives:

```
                       Ordinary Least Squares Estimation
*******************************************************************************
 Dependent variable is DM3
 117 observations used for estimation from 1968Q3 to 1997Q3
*******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 C                         72.6649            244.6070             .29707[.767]
 LAMBT                    117.2140             28.6640             4.0892[.000]
 DM3(-1)                   .081582             .096559             .84489[.400]
 DM3(-2)                    .11356             .096418             1.1778[.242]
 DM3(-3)                  -.073332              .10371            -.70709[.481]
 DM3(-4)                    .40219              .10064             3.9964[.000]
 DM3(-5)                  .9469E-3              .11639          .0081354[.994]
 DM3(-6)                   .076589              .11832             .64730[.519]
 DM3(-7)                   .029385              .11986             .24516[.807]
 DM3(-8)                   -.49709              .12006            -4.1404[.000]
 DM3(-9)                    .42534              .13104             3.2459[.002]
 DM3(-10)                  -.46963              .11832            -3.9692[.000]
 DM3(-11)                 -.017022              .12360            -.13772[.891]
 DM3(-12)                   .41276              .12591             3.2782[.001]
 DM3(-13)                  -.36764              .12332            -2.9812[.004]
*******************************************************************************
 R-Squared                      .78501   R-Bar-Squared                   .75550
 S.E. of Regression             1860.8   F-stat.    F( 14, 102)   26.6028[.000]
 Mean of Dependent Variable     3014.0   S.D. of Dependent Variable      3763.3
 Residual Sum of Squares      3.53E+08   Equation Log-likelihood        -1038.9
 Akaike Info. Criterion        -1053.9   Schwarz Bayesian Criterion     -1074.6
 DW-statistic                   1.9701
*******************************************************************************


                               Diagnostic Tests
*******************************************************************************
*    Test Statistics  *        LM Version        *         F Version          *
*******************************************************************************
*                     *                          *                            *
* A:Serial Correlation*CHSQ(   4)=  13.5114[.009]*F(   4,  98)=   3.1987[.016]*
*                     *                          *                            *
* B:Functional Form   *CHSQ(   1)=   .22310[.637]*F(   1, 101)=   .19296[.661]*
*                     *                          *                            *
* C:Normality         *CHSQ(   2)=  72.2382[.000]*       Not applicable       *
*                     *                          *                            *
* D:Heteroscedasticity*CHSQ(   1)=  10.6655[.001]*F(   1, 115)=  11.5346[.001]*
*******************************************************************************
   A:Lagrange multiplier test of residual serial correlation
   B:Ramsey's RESET test using the square of the fitted values
   C:Based on a test of skewness and kurtosis of residuals
   D:Based on the regression of squared residuals on squared fitted values
```

- And:

```
                       Ordinary Least Squares Estimation
*******************************************************************************
 Dependent variable is DM3
 117 observations used for estimation from 1968Q3 to 1997Q3
*******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 C                          62.4133             239.6178            .26047[.795]
 LAMBT                     111.1418              25.8468            4.3000[.000]
 DM3(-1)                    .081521              .080442            1.0134[.313]
 DM3(-2)                     .13806              .083829            1.6469[.103]
 DM3(-4)                     .40574              .094189            4.3078[.000]
 DM3(-8)                    -.49268               .10911           -4.5156[.000]
 DM3(-9)                     .40444               .10345            3.9095[.000]
 DM3(-10)                   -.43492              .098781           -4.4029[.000]
 DM3(-12)                    .40715               .11309            3.6002[.000]
 DM3(-13)                   -.33830               .10606           -3.1898[.002]
*******************************************************************************
 R-Squared                     .78244   R-Bar-Squared                   .76414
 S.E. of Regression            1827.7   F-stat.    F(  9, 107)   42.7571[.000]
 Mean of Dependent Variable    3014.0   S.D. of Dependent Variable      3763.3
 Residual Sum of Squares     3.57E+08   Equation Log-likelihood        -1039.6
 Akaike Info. Criterion       -1049.6   Schwarz Bayesian Criterion     -1063.4
 DW-statistic                  1.9648
*******************************************************************************


                               Diagnostic Tests
*******************************************************************************
*    Test Statistics  *        LM Version        *         F Version          *
*******************************************************************************
*                     *                          *                            *
* A:Serial Correlation*CHSQ(   4)=   5.2854[.259]*F(   4, 103)=   1.2183[.308]*
*                     *                          *                            *
* B:Functional Form   *CHSQ(   1)= .0028727[.957]*F(   1, 106)= .0026027[.959]*
*                     *                          *                            *
* C:Normality         *CHSQ(   2)=  80.4360[.000]*       Not applicable       *
*                     *                          *                            *
* D:Heteroscedasticity*CHSQ(   1)=   8.6122[.003]*F(   1, 115)=   9.1376[.003]*
*******************************************************************************
   A:Lagrange multiplier test of residual serial correlation
   B:Ramsey's RESET test using the square of the fitted values
   C:Based on a test of skewness and kurtosis of residuals
   D:Based on the regression of squared residuals on squared fitted values
```

- Thus:

$$\underset{(239.6178)}{\gamma = 62.4133} = 0$$
$$\underset{(25.8468)}{\delta = 111.1418} \neq 0$$

- Hence: ln-transform verified.

1.2 SC Criterion

- Estimate:

$$\Delta Y_t = \gamma + \sum \beta_i \Delta Y_{t-i} + u_t$$
$$\Delta y_t = \mu + \sum \theta_i \Delta y_{t-i} + \varepsilon_t$$

- To obtain Schwarz-Bayesian criterion.
- Choose the transformation with the *larger SC* criterion.

- Consider:

$$\Delta\left(\frac{M3}{P}\right)_t = \gamma + \sum \beta_i \Delta\left(\frac{M3}{P}\right)_{t-i} + u_t$$

$$\Delta\ln\left(\frac{M3}{P}\right)_t = \mu + \sum \theta_i \Delta\ln\left(\frac{M3}{P}\right)_{t-i} + \varepsilon_t$$

- Gives:

```
                       Ordinary Least Squares Estimation
*******************************************************************************
 Dependent variable is DM3
 117 observations used for estimation from 1968Q3 to 1997Q3
*******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 C                        165.3344           257.0008             .64332[.521]
 DM3(-1)                    .23007            .078309             2.9379[.004]
 DM3(-2)                    .32303            .077558             4.1650[.000]
 DM3(-4)                    .53164            .096499             5.5093[.000]
 DM3(-8)                   -.52314             .11736            -4.4575[.000]
 DM3(-9)                    .52514             .10733             4.8927[.000]
 DM3(-10)                  -.33494             .10349            -3.2365[.002]
 DM3(-12)                   .60875             .11093             5.4875[.000]
 DM3(-13)                  -.29748             .11387            -2.6125[.010]
*******************************************************************************
 R-Squared                     .74484   R-Bar-Squared                   .72594
 S.E. of Regression            1970.1   F-stat.    F(  8, 108)   39.4084[.000]
 Mean of Dependent Variable    3014.0   S.D. of Dependent Variable      3763.3
 Residual Sum of Squares     4.19E+08   Equation Log-likelihood        -1048.9
 Akaike Info. Criterion       -1057.9   Schwarz Bayesian Criterion     -1070.3
 DW-statistic                  1.9762
*******************************************************************************


                               Diagnostic Tests
*******************************************************************************
*    Test Statistics  *        LM Version        *         F Version          *
*******************************************************************************
*                     *                          *                            *
* A:Serial Correlation*CHSQ(   4)=   6.1647[.187]*F(   4, 104)=   1.4461[.224]*
*                     *                          *                            *
* B:Functional Form   *CHSQ(   1)=   .28564[.593]*F(   1, 107)=   .26187[.610]*
*                     *                          *                            *
* C:Normality         *CHSQ(   2)=  31.0074[.000]*       Not applicable       *
*                     *                          *                            *
* D:Heteroscedasticity*CHSQ(   1)=  14.9678[.000]*F(   1, 115)=  16.8702[.000]*
*******************************************************************************
   A:Lagrange multiplier test of residual serial correlation
   B:Ramsey's RESET test using the square of the fitted values
   C:Based on a test of skewness and kurtosis of residuals
   D:Based on the regression of squared residuals on squared fitted values
```

- And:

```
                       Ordinary Least Squares Estimation
******************************************************************************
 Dependent variable is DLM3
 117 observations used for estimation from 1968Q3 to 1997Q3
******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 C                         .021382           .0068833            3.1063[.002]
 DLM3(-1)                   .11994            .093627            1.2810[.203]
 DLM3(-2)                   .14808            .093723            1.5799[.117]
 DLM3(-4)                   .23550            .096459            2.4415[.016]
 DLM3(-8)                  -.11895            .099531           -1.1951[.235]
 DLM3(-9)                  .039329            .093783            .41936[.676]
 DLM3(-10)                -.061249            .091167           -.67182[.503]
 DLM3(-12)                  .10055            .093256            1.0782[.283]
 DLM3(-13)                -.078852            .091112           -.86544[.389]
******************************************************************************
 R-Squared                    .12840   R-Bar-Squared                   .063840
 S.E. of Regression          .017487   F-stat.    F(  8, 108)    1.9888[.055]
 Mean of Dependent Variable  .034748   S.D. of Dependent Variable      .018074
 Residual Sum of Squares     .033026   Equation Log-likelihood        312.0828
 Akaike Info. Criterion     303.0828   Schwarz Bayesian Criterion     290.6530
 DW-statistic                 2.0086
******************************************************************************


                               Diagnostic Tests
******************************************************************************
*    Test Statistics  *        LM Version        *         F Version          *
******************************************************************************
*                     *                          *                            *
* A:Serial Correlation*CHSQ(   4)=   2.2718[.686]*F(   4, 104)=   .51483[.725]*
*                     *                          *                            *
* B:Functional Form   *CHSQ(   1)=   .38415[.535]*F(   1, 107)=   .35247[.554]*
*                     *                          *                            *
* C:Normality         *CHSQ(   2)=   1.3869[.500]*       Not applicable       *
*                     *                          *                            *
* D:Heteroscedasticity*CHSQ(   1)=  .028659[.866]*F(   1, 115)=  .028176[.867]*
******************************************************************************
   A:Lagrange multiplier test of residual serial correlation
   B:Ramsey's RESET test using the square of the fitted values
   C:Based on a test of skewness and kurtosis of residuals
   D:Based on the regression of squared residuals on squared fitted values
```

- Such that:

$$\Delta\left(\frac{M3}{P}\right)_t = \gamma + \sum \beta_i \Delta\left(\frac{M3}{P}\right)_{t-i} + u_t$$

$$\Longrightarrow SC = -1070.3$$

$$\Delta\ln\left(\frac{M3}{P}\right)_t = \mu + \sum \theta_i \Delta\ln\left(\frac{M3}{P}\right)_{t-i} + \varepsilon_t$$

$$\Longrightarrow SC = 290.6530$$

- Hence: ln-transform verified.

2. Testing for Stationarity: Unit Root Tests

Recall the central problem is that in:

$$y_t = \rho y_{t-1} + \varepsilon_t$$

$\rho = 1$

$\Longrightarrow$ non-stationarity

Here: test for $\rho = 1$.

Important points:

1. DGP
   (a) may include a time trend: stochastic or deterministic.
   (b) may be more complex than a simple $AR(1)$ process; may involve MA-terms.
2. Unit root tests have problems with their "power":
   (a) in small samples they are likely to **over**-accept the null-hypothesis of a unit root.
   I.e. standard unit root tests will **under**-reject the presence of a unit root.

3. Structural breaks: "standard" unit root tests lead to an **under**-rejection of the null of the presence of a unit root.
4. In the case of higher frequency (eg. quarterly) data: check for seasonal unit roots.

Overview:

1. Use of autocorrelation function.
2. Use of spectrum.
3. Standard Tests:
   (a) (Augmented) Dickey-Fuller.
   (b) Phillips-Perron
   (c) Perron
   (d) Seasonal unit roots

2.1 Autocorrelation Function

- The $l'th$ order autocorrelation coefficient R_l for a time series is given by:

$$R_l = \frac{C_l}{C_0}, \; l = 1, 2, ..., \frac{n}{3}$$

$$where\ C_l = n^{-1} \sum_{t=l+1}^{n} (x_t - \overline{x})(x_{t-l} - \overline{x})$$

$$and\ n \equiv \; sample\ size$$

- With approximate estimate of the standard error of R_l of:

$$\widehat{se(R_l)} \approx \left(n^{-1} \left[1 + 2 \sum_{j=1}^{l-1} R_j^2 \right] \right)^{\frac{1}{2}}, \; l = 1, 2, ..., \frac{n}{3}$$

- Hence:

  $C_0 \equiv$ variance

  $C_1 \equiv$ *co*variance between x_2 and x_1

  ....

  $C_l \equiv$ *co*variance between x_{l+1} and x_1

  $\Longrightarrow$ under stationarity, autocovariance between two values depends only on distance between them

- Box-Pierce and Ljung-Box statistics:
  - test for the significance of the autocorrelation coefficients of the time series *jointly*,
- Difference:
  - Ljung-Box corrects for degrees of freedom;
  - Box-Pierce does not.

    $\Longrightarrow$ Ljung-Box performs better in small samples

- The Box-Pierce Q-statistic of order p is given by:

$$Q = n \sum_{j=1}^{p} R_j^2 \overset{a}{\sim} \chi_p^2$$

- The Ljung-Box statistic of order p is obtained from:

$$Q^* = n\,(n+2) \sum_{j=1}^{p} \frac{R_j^2}{(n-j)} \overset{a}{\sim} \chi_p^2$$

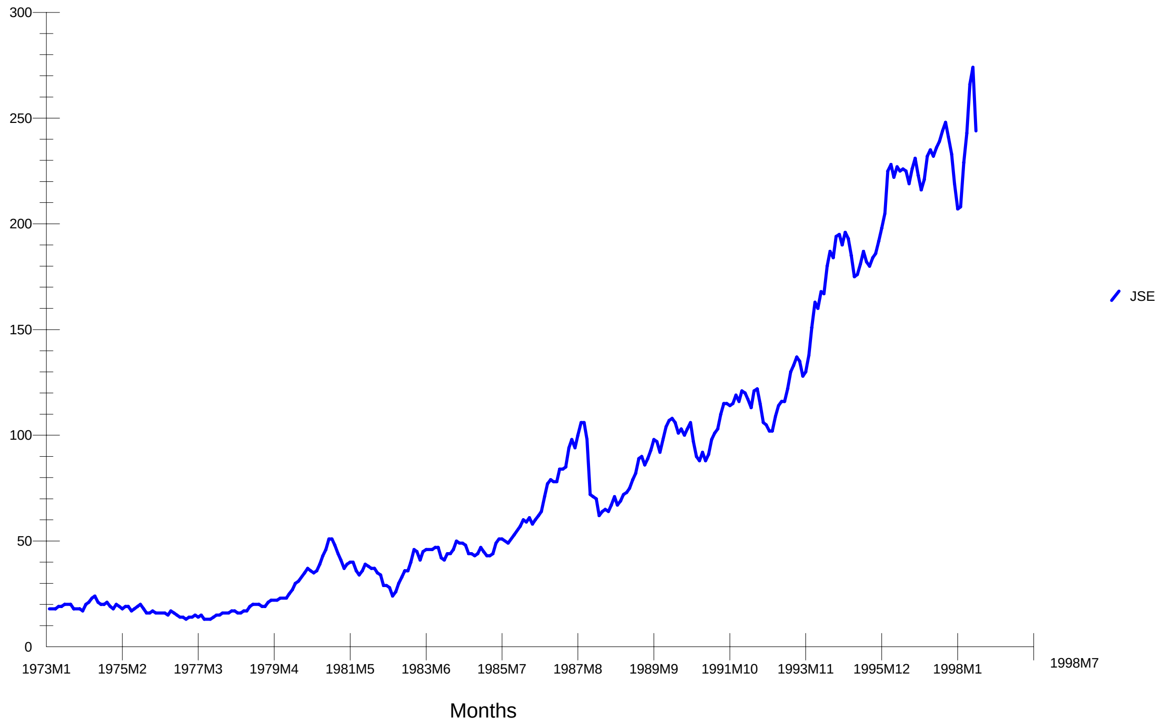
- Autocorrelation coefficients:
  - direct evidence on the nature of the ρ coefficient that has been central to our discussion of non-stationarity.
  - As long as the $R_l < 1$, we have sample evidence in favour of $\rho < 1$, hence stationarity.
  - Where $R_l = 1$ statistically, we have sample evidence in favour of $\rho = 1$, and hence non-stationarity.

- Note:
  - R_l will tend to underestimate ρ.
  - Kiewitz and Phillips (1989) show that:

$$R_l \approx \rho - \frac{k(t)}{t}$$

  - Bias will diminish as $t \rightarrow \infty$,
  - but $k(t)$ will also increase as $t \rightarrow \infty$.
  - Need: strong increase in t to eliminate the bias.
  - Given the size of most time series samples, therefore, the tendency will be to underestimate ρ.
  - Bootstrap: inadequate because themselves biased.

- EXAMPLE: Monthly Johannesburg Stock Exchange Index: 1973M1 1997M7

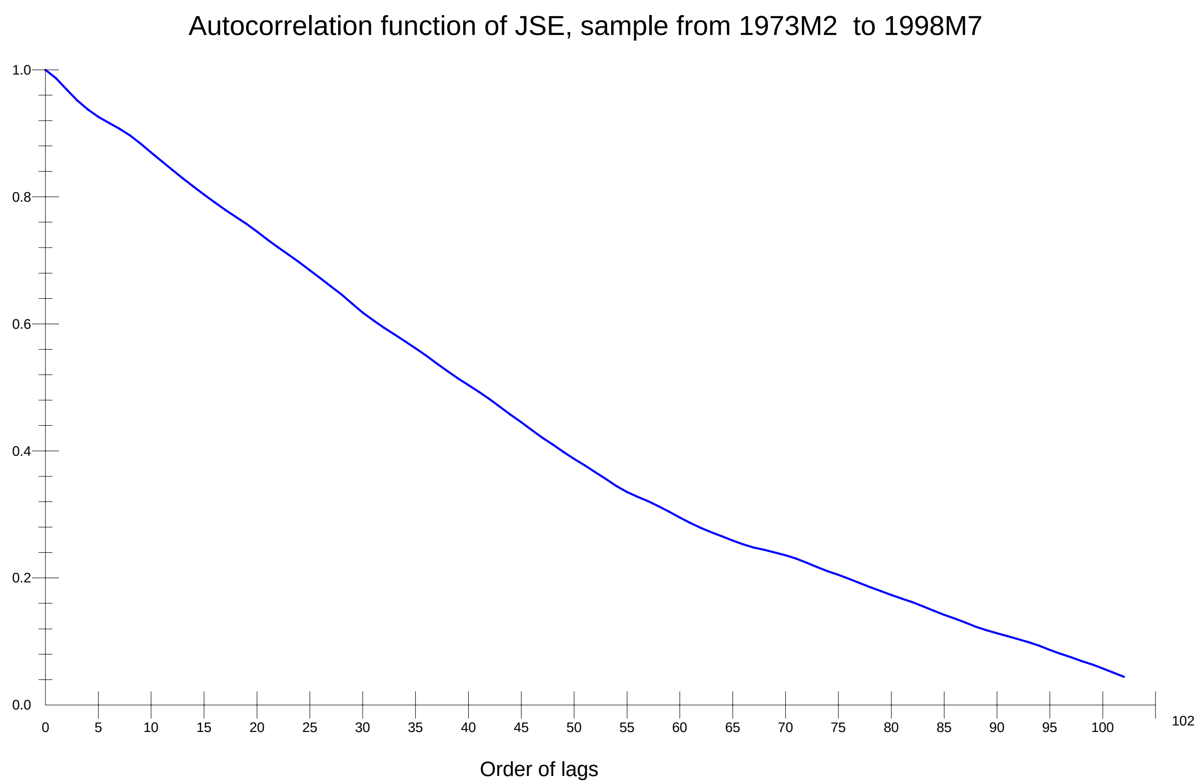


Gives:

Variable JSE Sample from 1973M2 to 1998M7

Order	Autocorrelation Coefficient	Standard Error	Box-Pierce Statistic	Ljung-Box Statistic
1	.98687	.057166	298.0201[.000]	300.9514[.000]
2	.96878	.098150	585.2093[.000]	591.9195[.000]
3	.95181	.12557	862.4253[.000]	873.7100[.000]
4	.93760	.14727	1131.4[.000]	1148.1[.000]
5	.92572	.16563	1393.7[.000]	1416.4[.000]
6	.91633	.18176	1650.6[.000]	1680.2[.000]
7	.90703	.19627	1902.3[.000]	1939.5[.000]
8	.89640	.20952	2148.2[.000]	2193.6[.000]
9	.88367	.22170	2387.2[.000]	2441.4[.000]
10	.87003	.23293	2618.8[.000]	2682.4[.000]
11	.85604	.24332	2843.0[.000]	2916.6[.000]
12	.84232	.25297	3060.1[.000]	3144.0[.000]
13	.82911	.26197	3270.5[.000]	3365.1[.000]
14	.81618	.27041	3474.3[.000]	3580.1[.000]
15	.80377	.27835	3672.0[.000]	3789.4[.000]
16	.79145	.28583	3863.7[.000]	3993.0[.000]
17	.77954	.29291	4049.7[.000]	4191.1[.000]
18	.76847	.29961	4230.4[.000]	4384.4[.000]
19	.75742	.30598	4405.9[.000]	4572.8[.000]
20	.74526	.31205	4575.9[.000]	4755.8[.000]
21	.73239	.31781	4740.0[.000]	4933.2[.000]
22	.72044	.32328	4898.8[.000]	5105.4[.000]
23	.70897	.32849	5052.6[.000]	5272.8[.000]
24	.69671	.33345	5201.2[.000]	5435.1[.000]
25	.68432	.33817	5344.5[.000]	5592.1[.000]
26	.67190	.34267	5482.6[.000]	5744.1[.000]
27	.65914	.34695	5615.6[.000]	5890.9[.000]
28	.64617	.35102	5743.3[.000]	6032.4[.000]
29	.63190	.35488	5865.5[.000]	6168.3[.000]
30	.61782	.35854	5982.3[.000]	6298.6[.000]

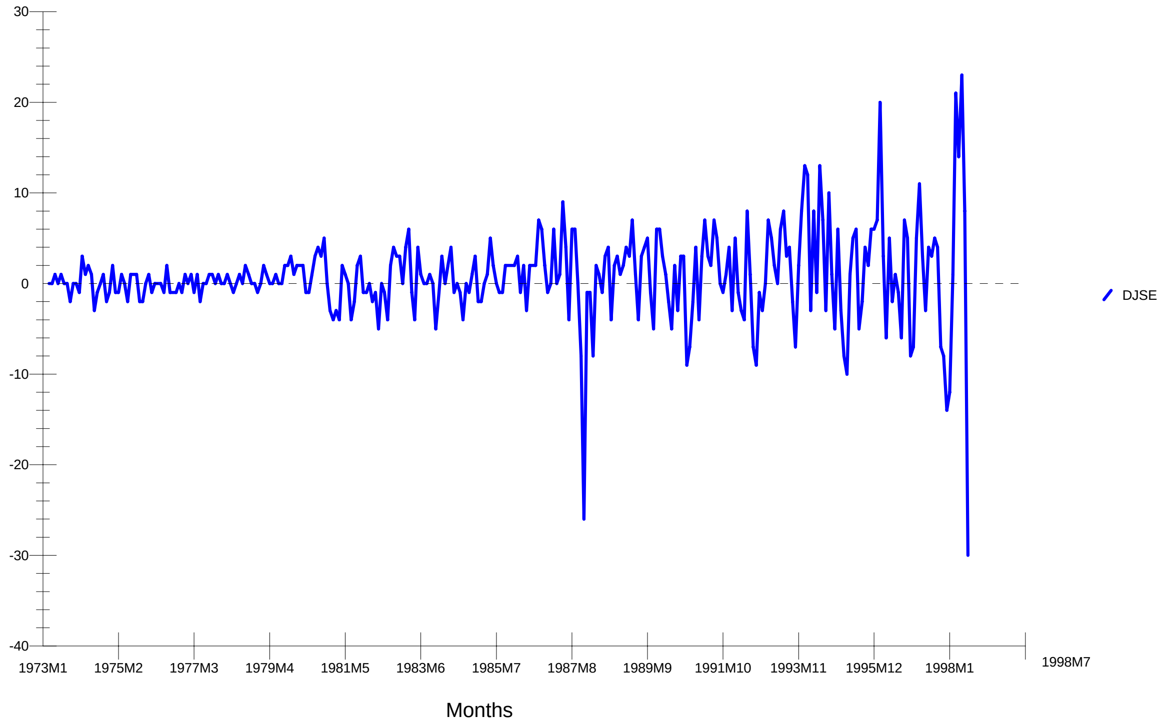
And:



Autocorrelation function of JSE, sample from 1973M2 to 1998M7
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
102
Order of lags

- Inference:
  - $\rho = 1$, throughout
  - JSE index nonstationary

• Now consider: First difference of JSE index

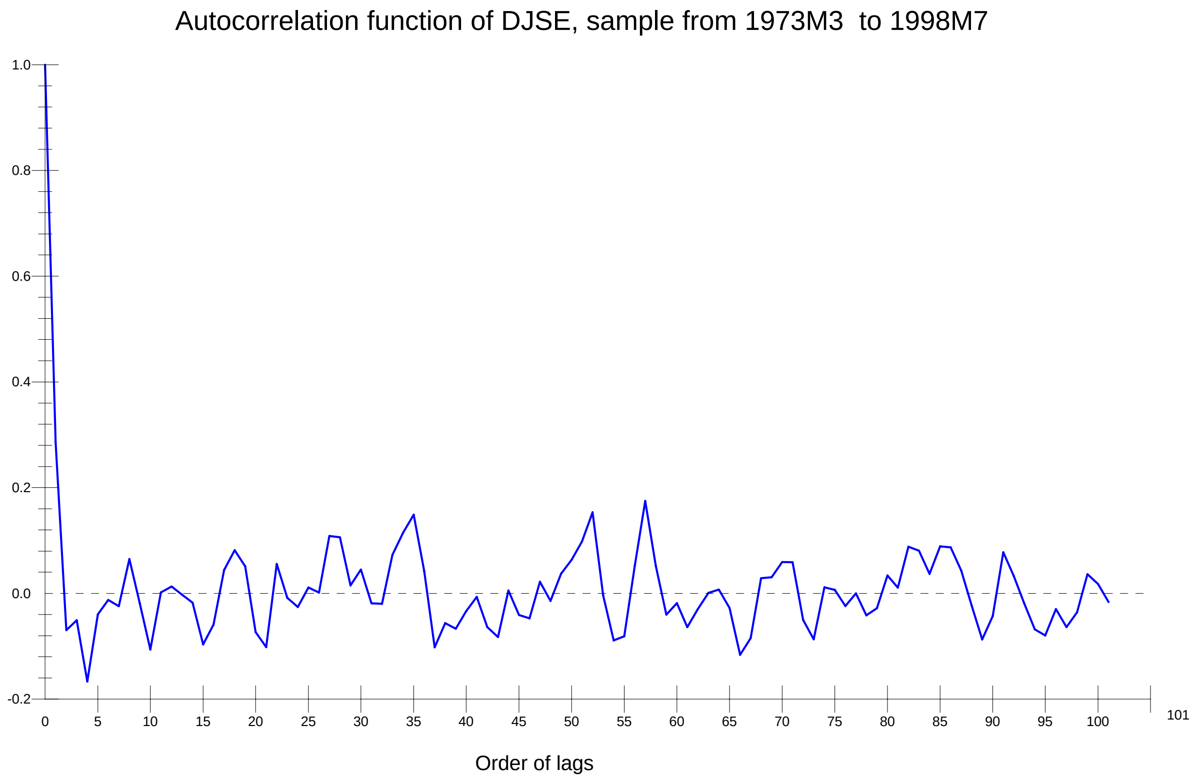


Gives:

Variable DJSE Sample from 1973M3 to 1998M7

Order	Autocorrelation Coefficient	Standard Error	Box-Pierce Statistic	Ljung-Box Statistic
1	.28585	.057260	24.9218[.000]	25.1677[.000]
2	-.069535	.061762	26.3965[.000]	26.6619[.000]
3	-.050860	.062018	27.1855[.000]	27.4639[.000]
4	-.16658	.062154	35.6493[.000]	36.0965[.000]
5	-.039676	.063601	36.1295[.000]	36.5879[.000]
6	-.012411	.063682	36.1764[.000]	36.6361[.000]
7	-.024370	.063690	36.3576[.000]	36.8227[.000]
8	.065229	.063721	37.6553[.000]	38.1641[.000]
9	-.018668	.063940	37.7616[.000]	38.2744[.000]
10	-.10617	.063957	41.1996[.000]	41.8522[.000]
11	.0015546	.064533	41.2003[.000]	41.8530[.000]
12	.012712	.064533	41.2496[.000]	41.9046[.000]
13	-.0027040	.064541	41.2519[.000]	41.9070[.000]
14	-.017792	.064541	41.3484[.000]	42.0088[.000]
15	-.096738	.064557	44.2027[.000]	45.0304[.000]
16	-.059125	.065031	45.2688[.000]	46.1630[.000]
17	.043997	.065207	45.8592[.000]	46.7923[.000]
18	.081439	.065304	47.8821[.000]	48.9562[.000]
19	.050894	.065636	48.6721[.000]	49.8042[.000]
20	-.073397	.065766	50.3152[.000]	51.5741[.000]
21	-.10151	.066034	53.4578[.000]	54.9713[.000]
22	.055598	.066543	54.4007[.000]	55.9940[.000]
23	-.0084986	.066695	54.4227[.000]	56.0180[.000]
24	-.025560	.066699	54.6220[.000]	56.2357[.000]
25	.011191	.066731	54.6602[.001]	56.2776[.000]
26	.0012694	.066737	54.6606[.001]	56.2781[.001]
27	.10833	.066737	58.2401[.000]	60.2310[.000]
28	.10605	.067311	61.6705[.000]	64.0329[.000]
29	.015000	.067857	61.7391[.000]	64.1092[.000]
30	.044885	.067868	62.3536[.000]	64.7952[.000]

And:


Autocorrelation function of DJSE, sample from 1973M3 to 1998M7
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
101
Order of lags

- Inference:
  - $\rho < 1$, throughout
  - DJSE index stationary

2.2 Spectrum

- Standardized spectral density function of a time series given by:

$$
\begin{aligned}
\widehat{f}\left(\omega_j\right) &= \lambda_0 + 2\sum_{k=1}^{m} \lambda_k R_k \cos\left(k\omega_j\right) \\
\omega_j &= \frac{j\pi}{m} \\
j &= 1, 2, ..., m \\
m &\equiv \textit{window size} \\
\lambda_k &\equiv \textit{set of lag weights} \\
&= \text{“}\textit{lag window}\text{]}
\end{aligned}
$$

- R_k denoting the autocorrelation coefficient of order k, defined by:

$$R_k = \frac{\sum_{t=k+1}^{n} (x_t - \overline{x})(x_{t-k} - \overline{x})}{\sum_{t=1}^{n} (x_t - \overline{x})^2}$$

- Alternative estimates of $\widehat{f}(\omega_j)$ from alternative lag windows. Eg.'s:

$$Bartlett\ Window: \quad \lambda_k = 1 - \frac{k}{m}\ ,\ 0 \le k \le m \tag{1}$$

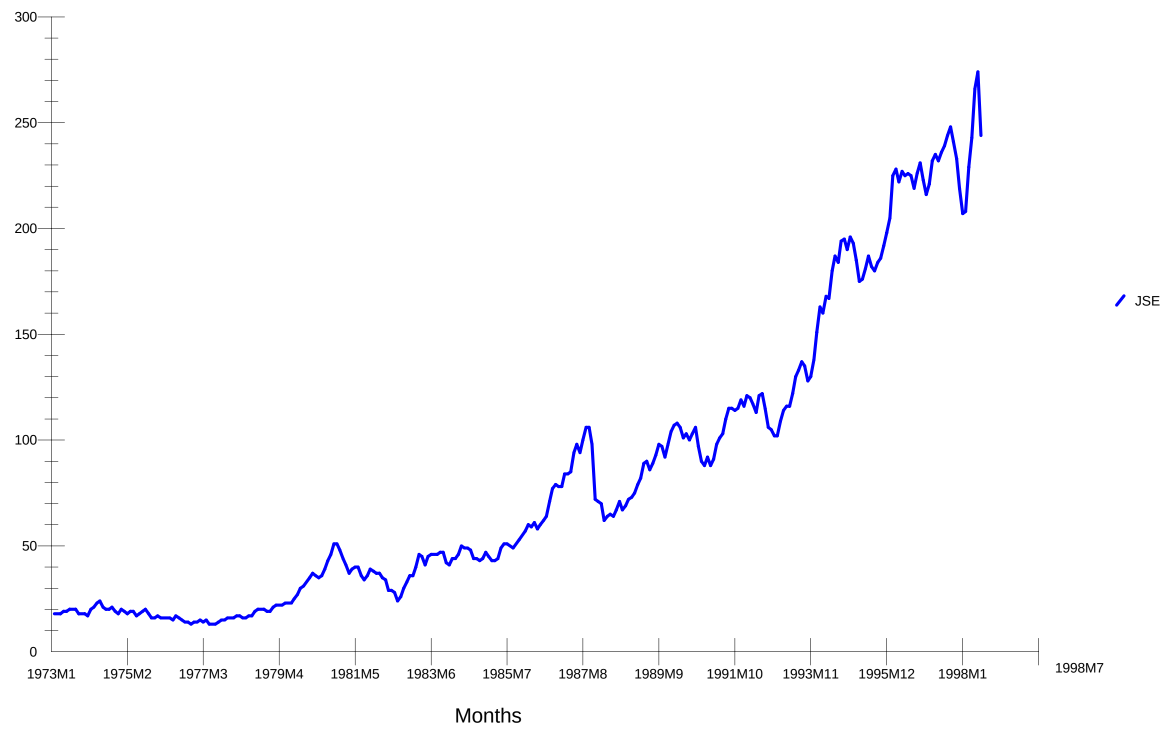
$$Tukey\ Window: \quad \lambda_k = \frac{1}{2}\left[1 + \cos\left(\frac{\pi k}{m}\right)\right]\ ,\ 0 \le k \le m$$

$$Parzen\ Window: \quad \lambda_k = \left\{ \begin{array}{l} 1 - 6\left(\frac{k}{m}\right)^2 + 6\left(\frac{k}{m}\right)^3\ ,\ 0 \le k \le \frac{m}{2} \\ 2\left(1 - \frac{k}{m}\right)^3\ ,\ \frac{m}{2} \le k \le m \end{array} \right\}$$

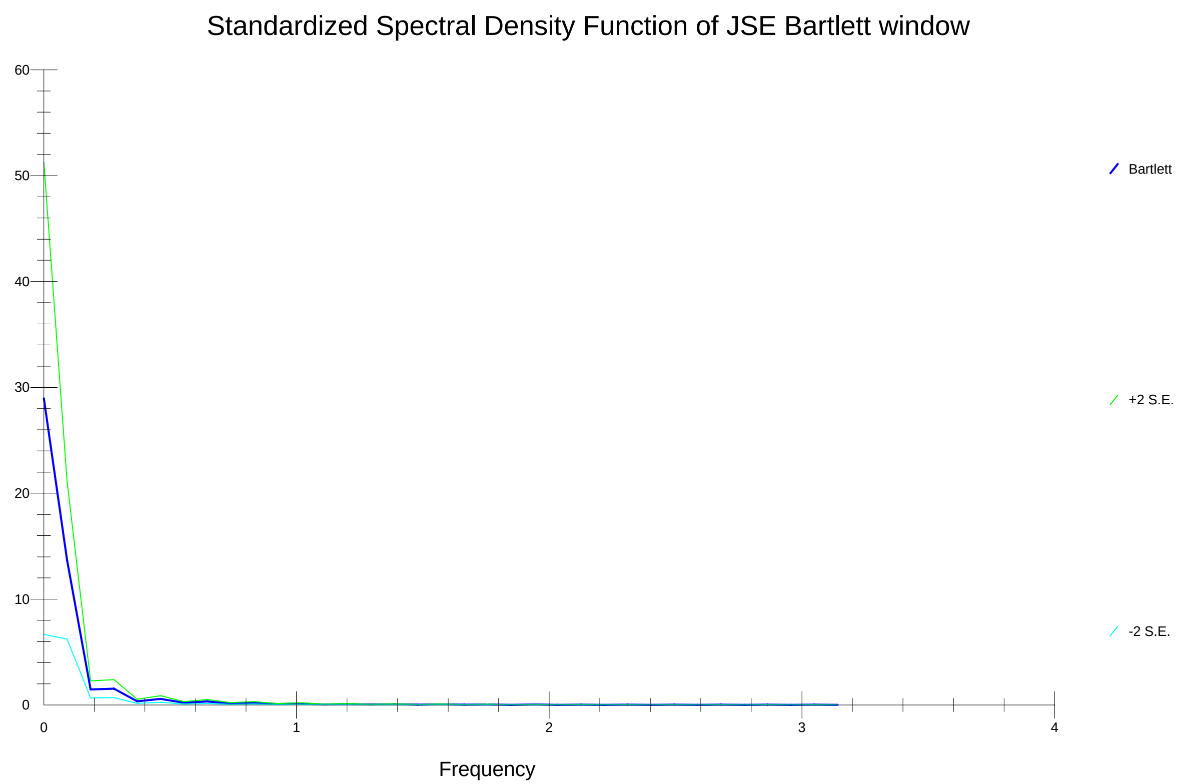
- Default value for $m = 2\sqrt{n}$.

- Intuition:
  - Under stationarity:
    - frequency, $\widehat{f}(\omega_j) \to \infty$
    - i.e. "frequently" see any given value of the series
  - Under non-stationarity:
    - frequency, $\widehat{f}(\omega_j) \to 0$
    - i.e. "infrequently" see any given value of the series

- EXAMPLE: Monthly Johannesburg Stock Exchange Index: 1973M1 1997M7

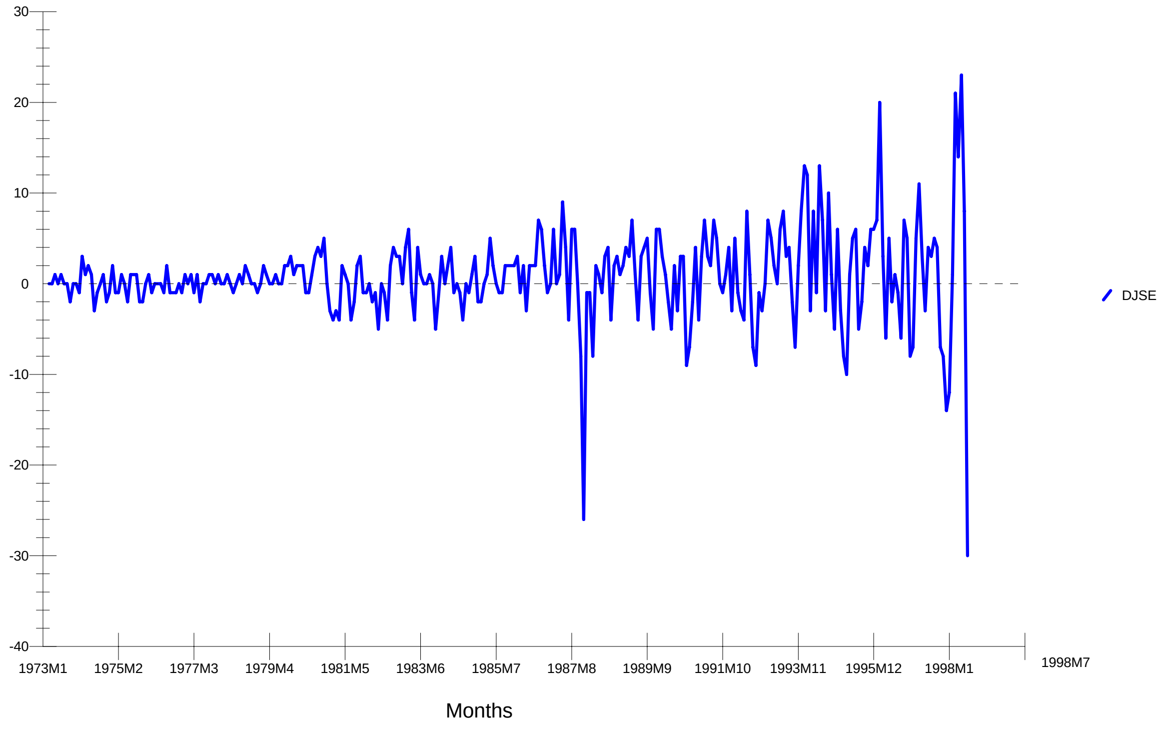


Gives:

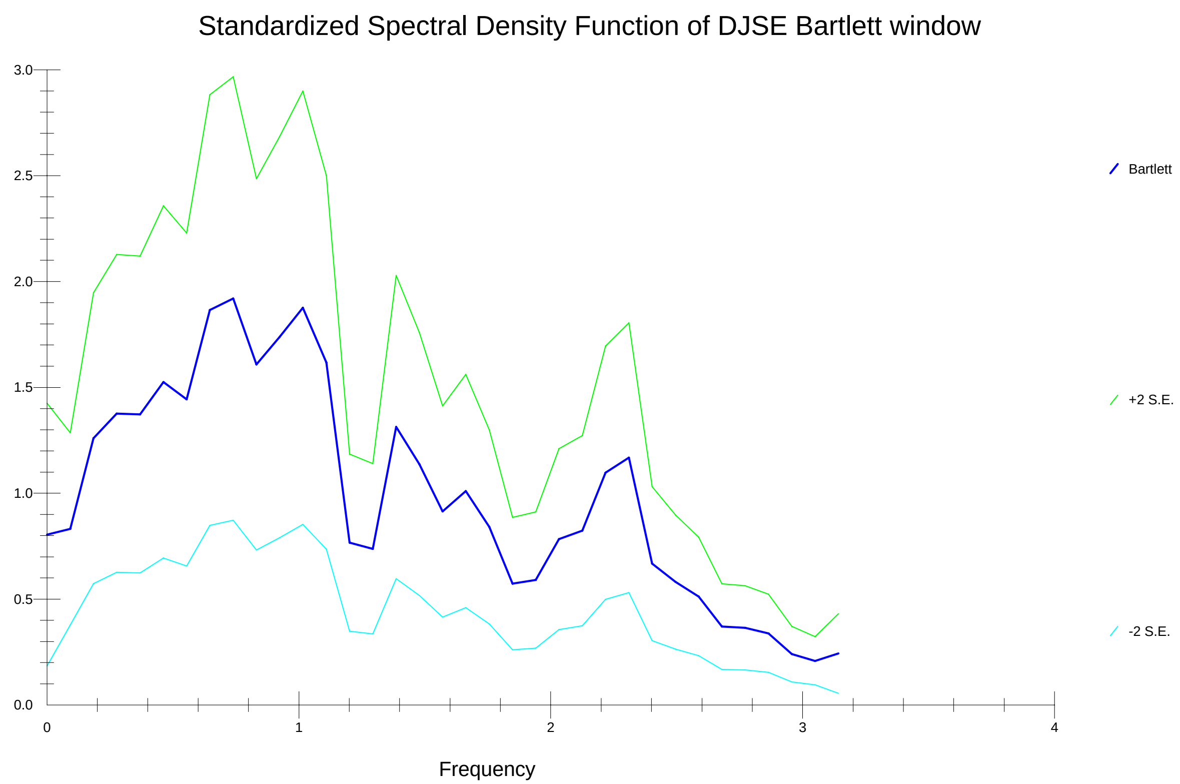

Standardized Spectral Density Function of JSE Bartlett window
60
50
40
30
20
10
0
0
1
2
3
4
Frequency
Bartlett
+2 S.E.
-2 S.E.

- Inference:
  - Most likely frequency =0
  - Hence: non-stationary

• Now consider: First difference of JSE index



Gives:



Standardized Spectral Density Function of DJSE Bartlett window
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
1
2
3
4
Frequency
Bartlett
+2 S.E.
-2 S.E.

- Inference:
  - Most likely frequency >0
  - Hence: stationary

2.3 Dickey-Fuller Tests

- Most commonly used tests.
- Popularity rests on:
  - their generality
  - their simplicity
- The DF test:
  - tests directly whether $\rho = 1$
  - adjusts t-statistics: non-standard
- 3 forms of $\tau-$test
- Plus 2 additional $\Phi-$tests
- Requires test sequence

2.3.1 No deterministic components: no mean, no trend: τ

- The DF-test estimates DGP of the form:

$$y_t = \rho_a y_{t-1} + u_t$$

or:

$$(1 - L)\, y_t = \Delta y_t = (\rho_a - 1)\, y_{t-1} + u_t \ , \ u_t \sim IID\left(0, \sigma^2\right)$$

with:

$$H_0 : \rho_a = 1$$
$$H_1 : \rho_a < 1$$

- The advantage of the second formulation, is that are testing:

$$\begin{aligned}
(\rho_a - 1) &= 0 \\
&against \\
(\rho_a - 1) &< 0
\end{aligned}$$

- Critical values are from Dickey and Fuller (1981:1063).

2.3.2 Deterministic components: mean, no trend: τ_μ

- τ-form of DF test:
  - presumes DGP has zero mean,
  - and no trend:

    $\Longrightarrow$ no deterministic components to the series.
  - Implication: initial y value is zero, since a DGP without deterministic components and a unit root has its mean determined by its initial observation.

- Where the mean of a nonstationary series is not zero, therefore, use of the initial version of the DF test would be inappropriate.
- Nankervis and Savin (1985): where the initial value of y is non-zero, τ-test leads to over-rejection of the null of nonstationarity.

- Hence τ_μ-test:

$$y_t = \mu_b + \rho_b y_{t-1} + u_t \ , \ u_t \sim IID\left(0, \sigma^2\right)$$

or:

$$(1-L)\, y_t = \Delta y_t = \mu_b + (\rho_b - 1)\, y_{t-1} + u_t \ , \ u_t \sim IID\left(0, \sigma^2\right)$$

with:

$$\begin{aligned} H_0 &: \rho_b = 1 \\ H_1 &: \rho_b < 1 \end{aligned}$$

- Critical values are from Dickey and Fuller (1981:1063).

2.3.3 Deterministic components: mean, trend:

τ_τ

- Suppose DGP was a trend-stationary process:

$$y_t = \alpha + \beta t + u_t$$

- Since the DGP contains a trend component, the second version of the DF test would be capable of fitting the trend only if it set $\rho_b = 1$.
- I.e. it finds the presence of a stochastic rather than a deterministic trend.

  $\Longrightarrow$ The first and second versions of the DF test effectively fail to distinguish between perhaps the two most commonly encountered time series: series with a stochastic and series with a deterministic trend.

- Hence employ version of the DF test which allows for the presence of a time trend, $\tau_{\mathcal{T}}$-test:

  $$y_t = \mu_c + \gamma_c t + \rho_c y_{t-1} + u_t \ , \ u_t \sim IID\left(0, \sigma^2\right)$$

  or:

  $$\Delta y_t = \mu_c + \gamma_c t + (\rho_c - 1)\, y_{t-1} + u_t \ , \ u_t \sim IID\left(0, \sigma^2\right)$$

- Note that: $\tau_{\mathcal{T}} < \tau_{\mu} < \tau$,
  $\Longrightarrow$ test becomes progressively more "demanding"

- Critical values are from Dickey and Fuller (1981:1063).

- There are two points to note here, however:
  - Where τ_τ-test is inappropriately **avoided**,
    $\Longrightarrow$ **over**-accept stationarity.
  - Where τ_τ-test is inappropriately **used**,
    $\Longrightarrow$ **under**-accept stationarity.

2.3.4 Augmented Dickey-Fuller Test Statistics

- DF tests thus far: $AR(1)$ processes.
- Where DGP follows $AR(p)$ process,
  $\Longrightarrow$ standard DF test critical values invalidated,
  $\Longrightarrow$ error term in the tests autocorrelated,
  $\Longrightarrow$ violate the "white noise" assumption of the DF tests.

- Correct for the possibility of autocorrelation: use *augmented DF* tests, or *ADF* tests:

$$\Delta y_t = \mu + \beta t + \vartheta_1 y_{t-1} + \sum_{i=1}^{p} \theta_i \Delta y_{t-i} + u_t$$

- Critical choice concerns the number of lags introduced into the *ADF* test:
  - Too few lags: over-rejection of the null of non-stationarity
  - Too many lags: reduces power of test.
- Convention: err on the side of generosity in the lag structure.

2.3.5 Dickey-Fuller Φ_3 Test: $\gamma_c = 0,\ \rho_c = 1$

- Testing restrictions on single coefficients by means of standard t-statistics is inappropriate in the presence of nonstationary variables.
- Testing joint restrictions by means of standard F-statistics is inappropriate in the presence of nonstationary variables.
- The Dickey-Fuller Φ_3 test tests the joint restriction that:

$$H_0 : \rho_c - 1 = \gamma_c = 0$$

- Critical values are from Dickey and Fuller (1981:1063).

2.3.6 Dickey-Fuller Φ_1 Test: $\mu_b = 0$, $\rho_b = 1$

- Φ_1 test tests the joint restriction that:

$$H_0 : \rho_b - 1 = \mu_b = 0$$

- Critical values are from Dickey and Fuller (1979), Table IV.

2.3.7 Perron Test Sequence

- Generalizes to ADF context.
- Perron suggests the following testing sequence:

1. τ_τ :

$$\begin{aligned}\Delta y_t &= \mu_c + \gamma_c t + (\rho_c - 1)\, y_{t-1} + u_t \\ H_0 &: \rho_c - 1 = 0,\ test\ statistic : \tau_\tau\end{aligned}$$

2. Φ_3 :

$$\begin{aligned}\Delta y_t &= \mu_c + \gamma_c t + (\rho_c - 1)\, y_{t-1} + u_t \\ H_0 &: \rho_c - 1 = \gamma_c = 0,\ test\ statistic : \Phi_3\end{aligned}$$

(a) t-statistic:

$$\begin{aligned}\Delta y_t &= \mu_c + \gamma_c t + (\rho_c - 1)\, y_{t-1} + u_t \\ H_0 &: \rho_c - 1 = 0,\ test\ statistic : t\end{aligned}$$

(b) For Φ_3 and Φ_1 tests, if null

$$H_0 : \rho_c = 1$$
$$H_0 : \rho_b = 1$$

not rejected, but the joint null hypotheses:

$$H_0 : \rho_c - 1 = \gamma_c = 0$$
$$H_0 : \rho_b - 1 = \mu_b = 0$$

is rejected, then:

i. the trend/drift is significant under the null of a unit root

ii. *asymptotic* normality of the t-statistic for ρ_i follows

3. τ_μ :

$$\begin{aligned}\Delta y_t &= \mu_b + (\rho_b - 1)\, y_{t-1} + u_t \\ H_0 &: \rho_b - 1 = 0,\ test\ statistic : \tau_\mu\end{aligned}$$

4. Φ_1 :

$$\begin{aligned}\Delta y_t &= \mu_b + (\rho_b - 1)\, y_{t-1} + u_t \\ H_0 &: \rho_b - 1 = \mu_b = 0,\ test\ statistic : \Phi_1\end{aligned}$$

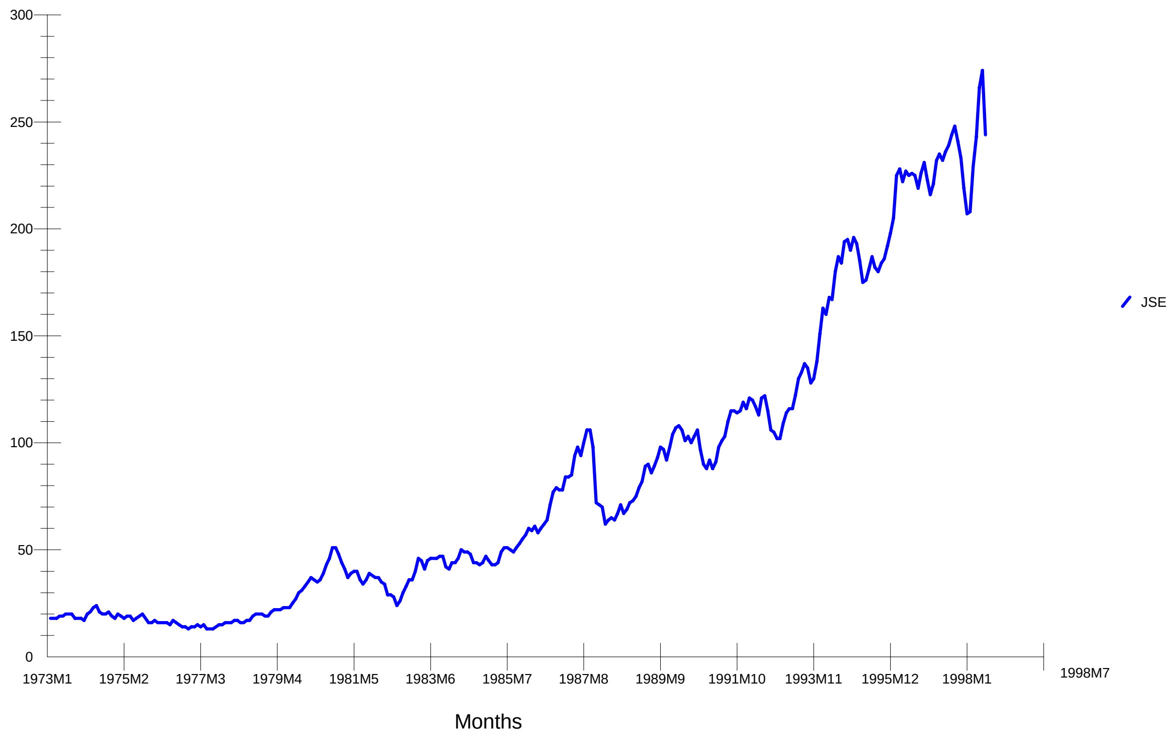
(a) t-statistic:

$$\begin{aligned}\Delta y_t &= \mu_b + (\rho_b - 1)\, y_{t-1} + u_t \\ H_0 &: \rho_b - 1 = 0,\ test\ statistic : t\end{aligned}$$

5. τ :

$$\begin{aligned}\Delta y_t &= (\rho_a - 1)\, y_{t-1} + u_t \\ H_0 &: \rho_a - 1 = 0,\ test\ statistic : \tau\end{aligned}$$

- EXAMPLE:Monthly Johannesburg Stock Exchange Index: 1973M1 1997M7



- Gives τ_τ:

```
                     Unit root tests for variable JSE
     The Dickey-Fuller regressions include an intercept and a linear trend
******************************************************************************
 293 observations used in the estimation of all ADF regressions.
 Sample period from 1974M3  to 1998M7
******************************************************************************
        Test Statistic      LL          AIC          SBC          HQC
 DF         -1.5333     -888.6219    -891.6219    -897.1422    -893.8329
 ADF(1)     -2.4641     -873.4903    -877.4903    -884.8506    -880.4382
 ADF(2)     -1.9207     -867.5362    -872.5362    -881.7367    -876.2211
 ADF(3)     -1.9334     -867.5049    -873.5049    -884.5454    -877.9268
 ADF(4)     -1.5580     -860.3410    -867.3410    -880.2216    -872.4998
 ADF(5)     -1.6303     -859.6836    -867.6836    -882.4043    -873.5794
 ADF(6)     -1.5315     -858.9270    -867.9270    -884.4878    -874.5598
 ADF(7)     -1.5624     -858.8211    -868.8211    -887.2220    -876.1909
 ADF(8)     -1.7328     -857.5435    -868.5435    -888.7845    -876.6503
 ADF(9)     -1.5399     -856.4118    -868.4118    -890.4928    -877.2555
 ADF(10)    -1.3354     -855.4353    -868.4353    -892.3565    -878.0161
 ADF(11)    -1.3724     -855.3549    -869.3549    -895.1161    -879.6727
 ADF(12)    -1.3043     -855.2338    -870.2338    -897.8351    -881.2885
******************************************************************************
 95% critical value for the augmented Dickey-Fuller statistic =  -3.4266
 LL  = Maximized log-likelihood      AIC = Akaike Information Criterion
 SBC = Schwarz Bayesian Criterion    HQC = Hannan-Quinn Criterion
```

- Inference: $H_0 : \rho_c - 1 = 0$ is not rejected,

- And Φ_3:

```
                       Ordinary Least Squares Estimation
*******************************************************************************
 Dependent variable is DJSE
 301 observations used for estimation from 1973M7  to 1998M7
*******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 C                         -.64842             .62177            -1.0429[.298]
 T                         .016307           .0077096             2.1152[.035]
 JSE(-1)                  -.014581            .010035            -1.4530[.147]
 DJSE(-1)                   .41530            .061405             6.7633[.000]
 DJSE(-2)                  -.26168            .066133            -3.9569[.000]
 DJSE(-3)                  .099438            .067373             1.4759[.141]
 DJSE(-4)                  -.24655            .063867            -3.8604[.000]
*******************************************************************************
 R-Squared                    .19072   R-Bar-Squared                    .17421
 S.E. of Regression           4.5620   F-stat.    F(  6, 294)   11.5479[.000]
 Mean of Dependent Variable   .74751   S.D. of Dependent Variable      5.0202
 Residual Sum of Squares      6118.8   Equation Log-likelihood      -880.4078
 Akaike Info. Criterion    -887.4078   Schwarz Bayesian Criterion   -900.3827
 DW-statistic                 1.8779
*******************************************************************************


                               Diagnostic Tests
*******************************************************************************
*    Test Statistics  *        LM Version        *         F Version          *
*******************************************************************************
*                     *                          *                            *
* A:Serial Correlation*CHSQ(  12)=  13.1649[.357]*F(  12, 282)=   1.0748[.381]*
*                     *                          *                            *
* B:Functional Form   *CHSQ(   1)=  .057116[.811]*F(   1, 293)=  .055609[.814]*
*                     *                          *                            *
* C:Normality         *CHSQ(   2)= 522.8766[.000]*       Not applicable       *
*                     *                          *                            *
* D:Heteroscedasticity*CHSQ(   1)=  12.7291[.000]*F(   1, 299)=  13.2029[.000]*
*******************************************************************************
   A:Lagrange multiplier test of residual serial correlation
   B:Ramsey's RESET test using the square of the fitted values
   C:Based on a test of skewness and kurtosis of residuals
   D:Based on the regression of squared residuals on squared fitted values
```

```
                       Variable Deletion Test (OLS case)
*******************************************************************************
 Dependent variable is DJSE
 List of the variables deleted from the regression:
 T               JSE(-1)
 301 observations used for estimation from 1973M7  to 1998M7
*******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 C                          .70617             .27475             2.5703[.011]
 DJSE(-1)                   .42063            .061045             6.8905[.000]
 DJSE(-2)                  -.26458            .065908            -4.0143[.000]
 DJSE(-3)                   .10046            .067519             1.4879[.138]
 DJSE(-4)                  -.24553            .063688            -3.8552[.000]
*******************************************************************************
 Joint test of zero restrictions on the coefficients of deleted variables:
 Lagrange Multiplier Statistic     CHSQ( 2)=   5.9316[.052]
 Likelihood Ratio Statistic        CHSQ( 2)=   5.9909[.050]
 F Statistic                    F(  2, 294)=   2.9551[.054]
*******************************************************************************
```

- Critical value: ≈ 6.34
- Inference:
  - $\rho_c - 1 = \gamma_c = 0$, not rejected
  - Hence:
    - the trend is *not* significant under the null of a unit root
    - *asymptotic* normality of the t-statistic for ρ_i does not follow

- And τ_μ:

```
                    Unit root tests for variable JSE
     The Dickey-Fuller regressions include an intercept but not a trend
*******************************************************************************
 293 observations used in the estimation of all ADF regressions.
 Sample period from 1974M3  to 1998M7
*******************************************************************************
        Test Statistic      LL          AIC          SBC          HQC
 DF          1.1515     -890.9399    -892.9399    -896.6200    -894.4138
 ADF(1)      .10451     -877.1988    -880.1988    -885.7191    -882.4098
 ADF(2)      .67985     -870.3829    -874.3829    -881.7432    -877.3308
 ADF(3)      .66204     -870.3795    -875.3795    -884.5799    -879.0644
 ADF(4)      1.1445     -862.7645    -868.7645    -879.8050    -873.1864
 ADF(5)      1.0182     -862.1776    -869.1776    -882.0582    -874.3365
 ADF(6)      1.1594     -861.3330    -869.3330    -884.0536    -875.2288
 ADF(7)      1.0949     -861.2597    -870.2597    -886.8205    -876.8926
 ADF(8)      .84352     -860.1772    -870.1772    -888.5780    -877.5470
 ADF(9)      1.0998     -858.8446    -869.8446    -890.0856    -877.9514
 ADF(10)     1.3762     -857.6770    -869.6770    -891.7580    -878.5207
 ADF(11)     1.2923     -857.6354    -870.6354    -894.5566    -880.2162
 ADF(12)     1.3729     -857.4605    -871.4605    -897.2217    -881.7783
*******************************************************************************
 95% critical value for the augmented Dickey-Fuller statistic =  -2.8715
 LL  = Maximized log-likelihood      AIC = Akaike Information Criterion
 SBC = Schwarz Bayesian Criterion    HQC = Hannan-Quinn Criterion
```

- Inference: $H_0 : \rho_b - 1 = 0$ is not rejected,

- And Φ_1:

```
                       Ordinary Least Squares Estimation
*******************************************************************************
 Dependent variable is DJSE
 301 observations used for estimation from 1973M7  to 1998M7
*******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 C                          .33500             .41526              .80671[.420]
 JSE(-1)                  .0048459           .0040676              1.1914[.234]
 DJSE(-1)                   .40956            .061705              6.6375[.000]
 DJSE(-2)                  -.27348            .066285             -4.1259[.000]
 DJSE(-3)                  .093647            .067713              1.3830[.168]
 DJSE(-4)                  -.25484            .064121             -3.9744[.000]
*******************************************************************************
 R-Squared                     .17841   R-Bar-Squared                    .16448
 S.E. of Regression            4.5888   F-stat.    F(  5, 295)   12.8118[.000]
 Mean of Dependent Variable    .74751   S.D. of Dependent Variable       5.0202
 Residual Sum of Squares       6211.9   Equation Log-likelihood       -882.6809
 Akaike Info. Criterion     -888.6809   Schwarz Bayesian Criterion    -899.8022
 DW-statistic                  1.8744
*******************************************************************************


                               Diagnostic Tests
*******************************************************************************
*    Test Statistics  *        LM Version        *         F Version          *
*******************************************************************************
*                     *                          *                            *
* A:Serial Correlation*CHSQ(  12)=  14.5657[.266]*F(  12, 283)=   1.1993[.283]*
*                     *                          *                            *
* B:Functional Form   *CHSQ(   1)=   .12887[.720]*F(   1, 294)=   .12592[.723]*
*                     *                          *                            *
* C:Normality         *CHSQ(   2)= 514.4048[.000]*       Not applicable       *
*                     *                          *                            *
* D:Heteroscedasticity*CHSQ(   1)=  11.0766[.001]*F(   1, 299)=  11.4234[.001]*
*******************************************************************************
   A:Lagrange multiplier test of residual serial correlation
   B:Ramsey's RESET test using the square of the fitted values
   C:Based on a test of skewness and kurtosis of residuals
   D:Based on the regression of squared residuals on squared fitted values
```

```
                       Variable Deletion Test (OLS case)
*******************************************************************************
 Dependent variable is DJSE
 List of the variables deleted from the regression:
 C               JSE(-1)
 301 observations used for estimation from 1973M7  to 1998M7
*******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 DJSE(-1)                    .44091            .061102             7.2160[.000]
 DJSE(-2)                  -.25177            .066337            -3.7953[.000]
 DJSE(-3)                    .11351            .067960             1.6703[.096]
 DJSE(-4)                  -.22656            .063853            -3.5481[.000]
*******************************************************************************
 Joint test of zero restrictions on the coefficients of deleted variables:
 Lagrange Multiplier Statistic     CHSQ( 2)=   7.9810[.018]
 Likelihood Ratio Statistic        CHSQ( 2)=   8.0887[.018]
 F Statistic                    F(  2, 295)=   4.0175[.019]
*******************************************************************************
```

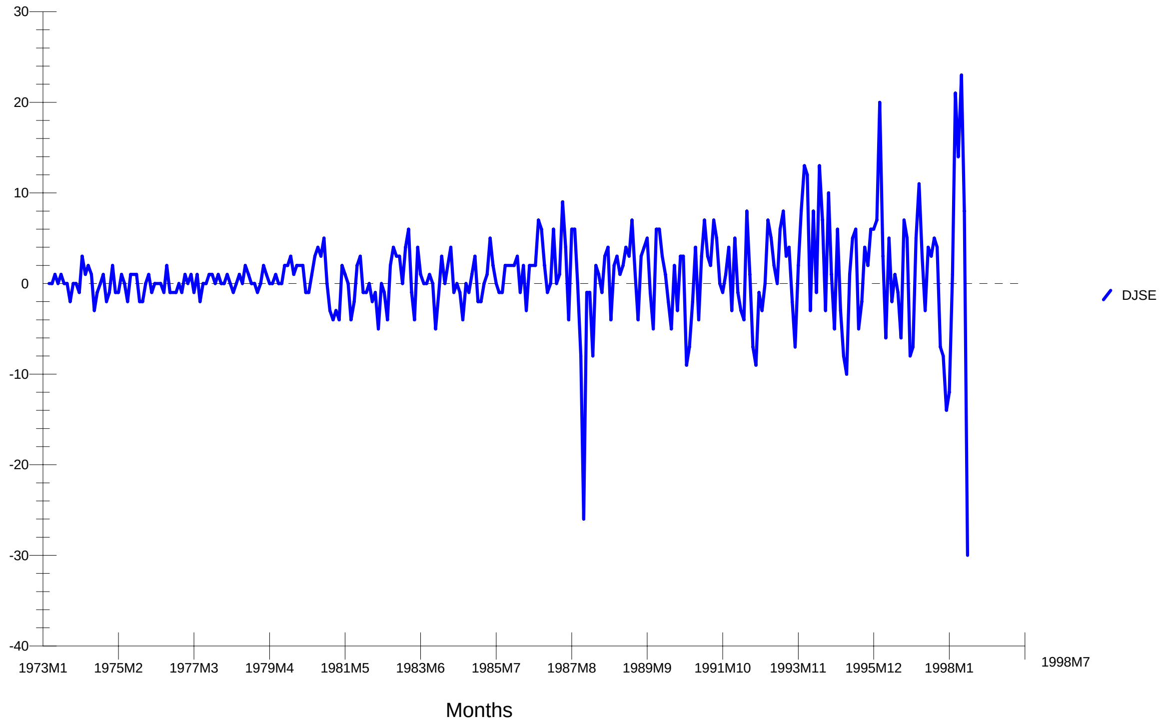
- Critical value: ≈ 4.63
- Inference:
  - $\rho_b - 1 = \mu_b = 0$, not rejected
  - Hence:
    - the drift is *not* significant under the null of a unit root
    - *asymptotic* normality of the t-statistic for ρ_i does not follow

- And τ:

```
                       Ordinary Least Squares Estimation
*******************************************************************************
 Dependent variable is DJSE
 301 observations used for estimation from 1973M7  to 1998M7
*******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 JSE(-1)                  .0073078          .0026877              2.7190[.007]
 DJSE(-1)                   .40815           .061644              6.6211[.000]
 DJSE(-2)                  -.27535           .066205             -4.1591[.000]
 DJSE(-3)                  .092890           .067667              1.3728[.171]
 DJSE(-4)                  -.25564           .064076             -3.9897[.000]
*******************************************************************************
 R-Squared                     .17660   R-Bar-Squared                   .16547
 S.E. of Regression            4.5861   F-stat.    F(  4, 296)   15.8707[.000]
 Mean of Dependent Variable    .74751   S.D. of Dependent Variable      5.0202
 Residual Sum of Squares       6225.6   Equation Log-likelihood      -883.0126
 Akaike Info. Criterion     -888.0126   Schwarz Bayesian Criterion   -897.2803
 DW-statistic                  1.8724
*******************************************************************************


                               Diagnostic Tests
*******************************************************************************
*    Test Statistics  *        LM Version        *         F Version          *
*******************************************************************************
*                     *                          *                            *
* A:Serial Correlation*CHSQ(  12)=  14.6988[.258]*F(  12, 284)=   1.2151[.272]*
*                     *                          *                            *
* B:Functional Form   *CHSQ(   1)=   .23549[.627]*F(   1, 295)=   .23097[.631]*
*                     *                          *                            *
* C:Normality         *CHSQ(   2)= 516.1714[.000]*       Not applicable       *
*                     *                          *                            *
* D:Heteroscedasticity*CHSQ(   1)=  10.4869[.001]*F(   1, 299)=  10.7933[.001]*
*******************************************************************************
   A:Lagrange multiplier test of residual serial correlation
   B:Ramsey's RESET test using the square of the fitted values
   C:Based on a test of skewness and kurtosis of residuals
   D:Based on the regression of squared residuals on squared fitted values
```

- Inference:
  - $\rho_a - 1 = 0$, not rejected
  - JSE nonstationary.

- Now consider: First difference of JSE index



- Gives τ_τ:

```
                       Unit root tests for variable DJSE
     The Dickey-Fuller regressions include an intercept and a linear trend
*******************************************************************************
 292 observations used in the estimation of all ADF regressions.
 Sample period from 1974M4  to 1998M7
*******************************************************************************
        Test Statistic      LL           AIC           SBC           HQC
 DF        -11.2958     -874.0434     -877.0434     -882.5585     -879.2525
 ADF(1)    -11.6444     -866.9308     -870.9308     -878.2843     -873.8763
 ADF(2)     -9.5632     -866.9305     -871.9305     -881.1224     -875.6124
 ADF(3)    -10.4857     -859.1350     -865.1350     -876.1653     -869.5533
 ADF(4)     -8.3173     -858.6026     -865.6026     -878.4712     -870.7572
 ADF(5)     -8.0488     -857.6947     -865.6947     -880.4017     -871.5858
 ADF(6)     -7.1139     -857.6423     -866.6423     -883.1877     -873.2697
 ADF(7)     -5.9863     -856.6592     -866.6592     -885.0430     -874.0230
 ADF(8)     -6.2128     -855.2108     -866.2108     -886.4330     -874.3110
 ADF(9)     -6.3615     -853.9412     -865.9412     -888.0018     -874.7778
 ADF(10)    -5.8121     -853.9165     -866.9165     -890.8154     -876.4895
 ADF(11)    -5.6796     -853.7043     -867.7043     -893.4416     -878.0136
 ADF(12)    -5.6130     -853.3948     -868.3948     -895.9704     -879.4405
*******************************************************************************
 95% critical value for the augmented Dickey-Fuller statistic =  -3.4266
 LL  = Maximized log-likelihood      AIC = Akaike Information Criterion
 SBC = Schwarz Bayesian Criterion    HQC = Hannan-Quinn Criterion
```

- Inference: $H_0 : \rho_c - 1 = 0$ is rejected,

- And Φ_3:

```
                       Ordinary Least Squares Estimation
*******************************************************************************
 Dependent variable is DDJSE
 300 observations used for estimation from 1973M8  to 1998M7
*******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 C                         -.22516             .54974            -.40958[.682]
 T                        .0058535           .0031462             1.8605[.064]
 DJSE(-1)                  -.96827             .11472            -8.4403[.000]
 DDJSE(-1)                  .38390             .10249             3.7458[.000]
 DDJSE(-2)                  .10414            .095354             1.0922[.276]
 DDJSE(-3)                  .21118            .077425             2.7275[.007]
 DDJSE(-4)                -.068543            .066674            -1.0280[.305]
*******************************************************************************
 R-Squared                     .37679   R-Bar-Squared                    .36403
 S.E. of Regression            4.5772   F-stat.    F(  6, 293)   29.5246[.000]
 Mean of Dependent Variable   -.10333   S.D. of Dependent Variable       5.7396
 Residual Sum of Squares       6138.5   Equation Log-likelihood       -878.4639
 Akaike Info. Criterion     -885.4639   Schwarz Bayesian Criterion    -898.4271
 DW-statistic                  1.8954
*******************************************************************************


                               Diagnostic Tests
*******************************************************************************
*    Test Statistics  *        LM Version        *         F Version          *
*******************************************************************************
*                     *                          *                            *
* A:Serial Correlation*CHSQ(  12)=  12.3098[.421]*F(  12, 281)=   1.0020[.447]*
*                     *                          *                            *
* B:Functional Form   *CHSQ(   1)=   3.9031[.048]*F(   1, 292)=   3.8491[.051]*
*                     *                          *                            *
* C:Normality         *CHSQ(   2)= 486.1393[.000]*       Not applicable       *
*                     *                          *                            *
* D:Heteroscedasticity*CHSQ(   1)=  31.0233[.000]*F(   1, 298)=  34.3708[.000]*
*******************************************************************************
   A:Lagrange multiplier test of residual serial correlation
   B:Ramsey's RESET test using the square of the fitted values
   C:Based on a test of skewness and kurtosis of residuals
   D:Based on the regression of squared residuals on squared fitted values
```

```
                      Variable Deletion Test (OLS case)
*******************************************************************************
 Dependent variable is DDJSE
 List of the variables deleted from the regression:
 C               T
 300 observations used for estimation from 1973M8  to 1998M7
*******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 DJSE(-1)                  -.83884            .10768            -7.7904[.000]
 DDJSE(-1)                  .29096           .098925             2.9412[.004]
 DDJSE(-2)                 .028381           .093092              .30487[.761]
 DDJSE(-3)                  .16183           .076584             2.1131[.035]
 DDJSE(-4)                 -.10174           .066544            -1.5289[.127]
*******************************************************************************
 Joint test of zero restrictions on the coefficients of deleted variables:
 Lagrange Multiplier Statistic     CHSQ( 2)=   9.0484[.011]
 Likelihood Ratio Statistic        CHSQ( 2)=   9.1876[.010]
 F Statistic                    F(  2, 293)=   4.5560[.011]
*******************************************************************************
```

- Critical value: ≈ 6.34
- Inference:
  - $\rho_c - 1 = \gamma_c = 0$, not rejected
  - Hence:
    - the trend is *not* significant under the null of a unit root
    - *asymptotic* normality of the t-statistic for ρ_i does not follow

- And τ_μ:

```
                       Unit root tests for variable DJSE
      The Dickey-Fuller regressions include an intercept but not a trend
*******************************************************************************
 292 observations used in the estimation of all ADF regressions.
 Sample period from 1974M4  to 1998M7
*******************************************************************************
        Test Statistic      LL          AIC          SBC          HQC
 DF        -11.2497     -874.7051    -876.7051    -880.3818    -878.1778
 ADF(1)    -11.5324     -868.1196    -871.1196    -876.6347    -873.3287
 ADF(2)     -9.4301     -868.1048    -872.1048    -879.4583    -875.0503
 ADF(3)    -10.2764     -860.9471    -865.9471    -875.1390    -869.6290
 ADF(4)     -8.0926     -860.2349    -866.2349    -877.2651    -870.6531
 ADF(5)     -7.7844     -859.5514    -866.5514    -879.4201    -871.7061
 ADF(6)     -6.8388     -859.4074    -867.4074    -882.1145    -873.2985
 ADF(7)     -5.7362     -858.0756    -867.0756    -883.6210    -873.7030
 ADF(8)     -5.9040     -856.9973    -866.9973    -885.3811    -874.3611
 ADF(9)     -5.9762     -856.1960    -867.1960    -887.4181    -875.2962
 ADF(10)    -5.4227     -856.0449    -868.0449    -890.1055    -876.8815
 ADF(11)    -5.2536     -855.9844    -868.9844    -892.8833    -878.5573
 ADF(12)    -5.1442     -855.8714    -869.8714    -895.6087    -880.1807
*******************************************************************************
 95% critical value for the augmented Dickey-Fuller statistic =  -2.8716
 LL  = Maximized log-likelihood      AIC = Akaike Information Criterion
 SBC = Schwarz Bayesian Criterion    HQC = Hannan-Quinn Criterion
```

- Inference: $H_0 : \rho_b - 1 = 0$ is rejected,

- And Φ_1:

```
                       Ordinary Least Squares Estimation
*******************************************************************************
 Dependent variable is DDJSE
 300 observations used for estimation from 1973M8  to 1998M7
*******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 C                           .65834              .27811            2.3672[.019]
 DJSE(-1)                  -.91762              .11191           -8.1998[.000]
 DDJSE(-1)                   .34842              .10112            3.4457[.001]
 DDJSE(-2)                  .076283             .094564            .80668[.421]
 DDJSE(-3)                   .19329             .077147            2.5055[.013]
 DDJSE(-4)                -.080081             .066662           -1.2013[.231]
*******************************************************************************
 R-Squared                      .36943   R-Bar-Squared                   .35871
 S.E. of Regression             4.5963   F-stat.    F(  5, 294)   34.4489[.000]
 Mean of Dependent Variable   -.10333   S.D. of Dependent Variable      5.7396
 Residual Sum of Squares       6211.0   Equation Log-likelihood      -880.2256
 Akaike Info. Criterion     -886.2256   Schwarz Bayesian Criterion   -897.3369
 DW-statistic                   1.8987
*******************************************************************************


                               Diagnostic Tests
*******************************************************************************
*    Test Statistics  *        LM Version        *         F Version          *
*******************************************************************************
*                     *                          *                            *
* A:Serial Correlation*CHSQ(  12)=  10.1276[.605]*F(  12, 282)=   .82104[.629]*
*                     *                          *                            *
* B:Functional Form   *CHSQ(   1)=   1.1845[.276]*F(   1, 293)=   1.1615[.282]*
*                     *                          *                            *
* C:Normality         *CHSQ(   2)= 471.9442[.000]*       Not applicable       *
*                     *                          *                            *
* D:Heteroscedasticity*CHSQ(   1)=  32.5907[.000]*F(   1, 298)=  36.3190[.000]*
*******************************************************************************
   A:Lagrange multiplier test of residual serial correlation
   B:Ramsey's RESET test using the square of the fitted values
   C:Based on a test of skewness and kurtosis of residuals
   D:Based on the regression of squared residuals on squared fitted values
```

Variable Deletion Test (OLS case)

****

Dependent variable is DDJSE

List of the variables deleted from the regression:

C DJSE(-1)

300 observations used for estimation from 1973M8 to 1998M7

****

Regressor	Coefficient	Standard Error	T-Ratio[Prob]
DDJSE(-1)	-.34569	.061110	-5.6568[.000]
DDJSE(-2)	-.52586	.065814	-7.9900[.000]
DDJSE(-3)	-.21112	.065528	-3.2218[.001]
DDJSE(-4)	-.36990	.062428	-5.9252[.000]

****

Joint test of zero restrictions on the coefficients of deleted variables:

Lagrange Multiplier Statistic	CHSQ(2)=	55.8420[.000]
Likelihood Ratio Statistic	CHSQ(2)=	61.7901[.000]
F Statistic	F(2, 294)=	33.6208[.000]

****

- Critical value: ≈ 4.63
- Inference:
  - $\rho_b - 1 = \mu_b = 0$, is rejected
  - Hence:
    - the drift *is* significant under the null of a unit root
      $\Longrightarrow$ invalidates τ test
    - *asymptotic* normality of the t-statistic for ρ_i does follow

- So:

```
                       Ordinary Least Squares Estimation
*******************************************************************************
 Dependent variable is DDJSE
 300 observations used for estimation from 1973M8  to 1998M7
*******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 C                          .65834             .27811            2.3672[.019]
 DJSE(-1)                  -.91762             .11191           -8.1998[.000]
 DDJSE(-1)                  .34842             .10112            3.4457[.001]
 DDJSE(-2)                 .076283            .094564            .80668[.421]
 DDJSE(-3)                  .19329            .077147            2.5055[.013]
 DDJSE(-4)                -.080081            .066662           -1.2013[.231]
*******************************************************************************
 R-Squared                     .36943   R-Bar-Squared                   .35871
 S.E. of Regression            4.5963   F-stat.    F(  5, 294)   34.4489[.000]
 Mean of Dependent Variable   -.10333   S.D. of Dependent Variable      5.7396
 Residual Sum of Squares       6211.0   Equation Log-likelihood      -880.2256
 Akaike Info. Criterion     -886.2256   Schwarz Bayesian Criterion   -897.3369
 DW-statistic                  1.8987
*******************************************************************************


                               Diagnostic Tests
*******************************************************************************
*    Test Statistics  *        LM Version        *         F Version          *
*******************************************************************************
*                     *                          *                            *
* A:Serial Correlation*CHSQ(  12)=  10.1276[.605]*F(  12, 282)=   .82104[.629]*
*                     *                          *                            *
* B:Functional Form   *CHSQ(   1)=   1.1845[.276]*F(   1, 293)=   1.1615[.282]*
*                     *                          *                            *
* C:Normality         *CHSQ(   2)= 471.9442[.000]*       Not applicable       *
*                     *                          *                            *
* D:Heteroscedasticity*CHSQ(   1)=  32.5907[.000]*F(   1, 298)=  36.3190[.000]*
*******************************************************************************
   A:Lagrange multiplier test of residual serial correlation
   B:Ramsey's RESET test using the square of the fitted values
   C:Based on a test of skewness and kurtosis of residuals
   D:Based on the regression of squared residuals on squared fitted values
```

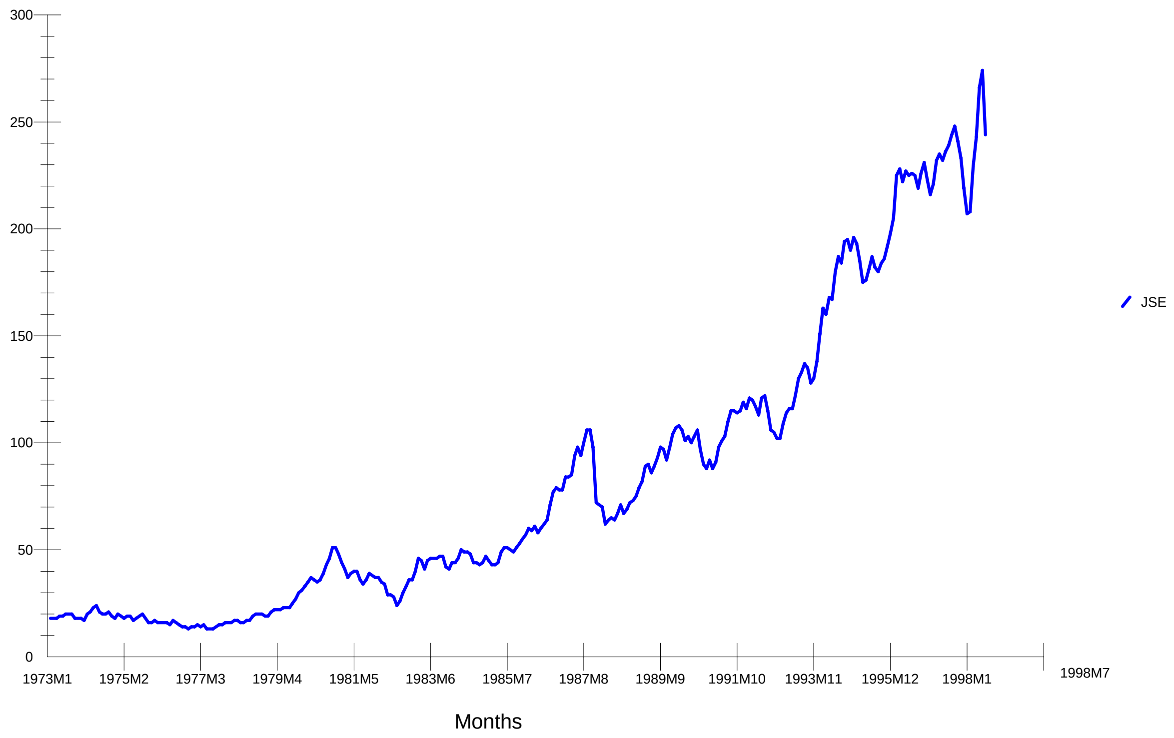
- Inference:
  - $-0.91762 < 0$

    $\Longrightarrow \rho < 1$
  - Stationary, subject to drift.

2.4 Phillips-Perron Test

- Alternative to ADF test
- Proposed by Phillips (1987), Perron (1988) and Phillips and Perron (1988).
- Standard DF tests.
- Undertaking a nonparametric correction to the t-test to “compensate” for the biasing effects of the autocorrelation.
- Critical values: ADF.
- Monte Carlo work (see Schwert 1989):
  - poor size properties,
  - reject null of nonstationarity too easily when DGP has large negative MA components.

- EXAMPLE: Monthly Johannesburg Stock Exchange Index: 1973M1 1997M7



Gives:

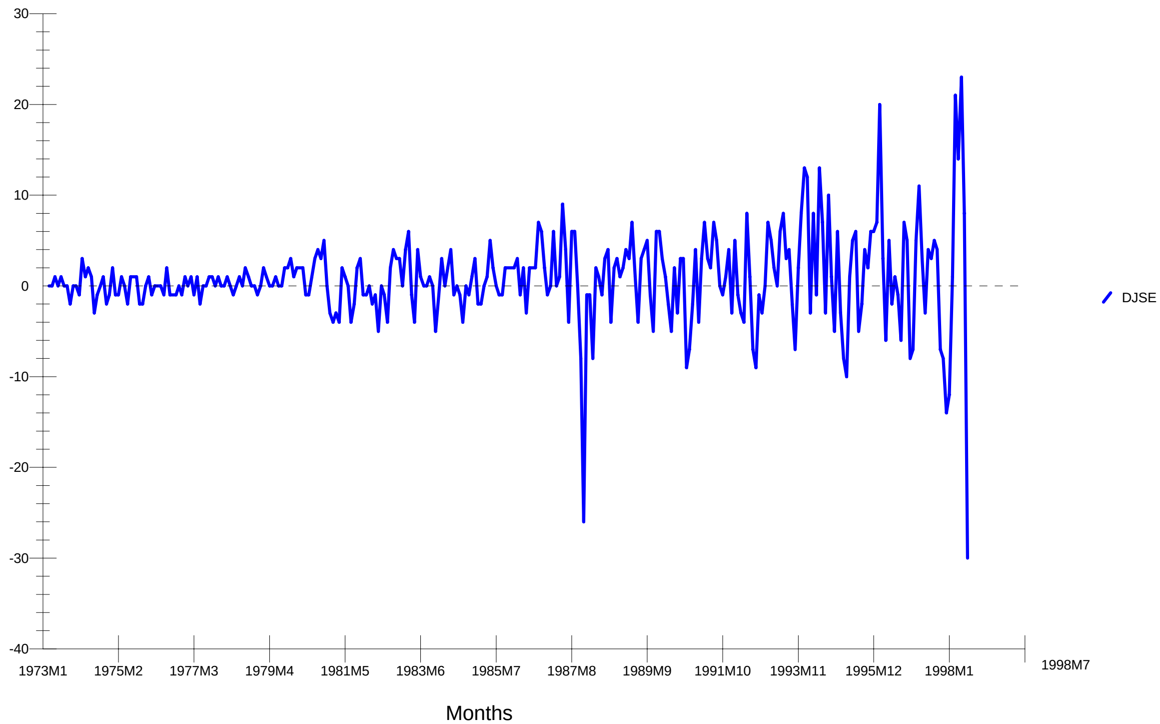
```
                       Ordinary Least Squares Estimation
*******************************************************************************
 Dependent variable is DJSE
 305 observations used for estimation from 1973M3  to 1998M7
*******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 C                           .32241            .44456             .72522[.469]
 JSE(-1)                   .0051296          .0041777             1.2279[.220]
*******************************************************************************
 R-Squared                    .0049511   R-Bar-Squared                 .0016671
 S.E. of Regression             4.9835   F-stat.    F(  1, 303)    1.5077[.220]
 Mean of Dependent Variable     .74098   S.D. of Dependent Variable      4.9877
 Residual Sum of Squares        7525.1   Equation Log-likelihood      -921.6435
 Akaike Info. Criterion      -923.6435   Schwarz Bayesian Criterion   -927.3638
 DW-statistic                   1.3159
*******************************************************************************


                               Diagnostic Tests
*******************************************************************************
*    Test Statistics  *        LM Version        *         F Version          *
*******************************************************************************
*                     *                          *                            *
* A:Serial Correlation*CHSQ(  12)=  62.0648[.000]*F(  12, 291)=   6.1954[.000]*
*                     *                          *                            *
* B:Functional Form   *CHSQ(   1)=   3.7203[.054]*F(   1, 302)=   3.7292[.054]*
*                     *                          *                            *
* C:Normality         *CHSQ(   2)=   1224.2[.000]*       Not applicable       *
*                     *                          *                            *
* D:Heteroscedasticity*CHSQ(   1)=  49.0522[.000]*F(   1, 303)=  58.0698[.000]*
*******************************************************************************
   A:Lagrange multiplier test of residual serial correlation
   B:Ramsey's RESET test using the square of the fitted values
   C:Based on a test of skewness and kurtosis of residuals
   D:Based on the regression of squared residuals on squared fitted values
```

And then:

```
                       Ordinary Least Squares Estimation
   Based on Newey-West adjusted S.E.'s Bartlett weights, truncation lag= 12
*******************************************************************************
 Dependent variable is DJSE
 305 observations used for estimation from 1973M3  to 1998M7
*******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 C                          .32241             .30000            1.0747[.283]
 JSE(-1)                  .0051296           .0043863            1.1695[.243]
*******************************************************************************
```

- Inference:
  - $\rho = 1$
  - Non-stationary

- Now consider: First difference of JSE index



Gives:

```
                      Ordinary Least Squares Estimation
*******************************************************************************
 Dependent variable is DDJSE
 304 observations used for estimation from 1973M4  to 1998M7
*******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 C                          .46821             .27768            1.6861[.093]
 DJSE(-1)                  -.67319            .058583          -11.4912[.000]
*******************************************************************************
 R-Squared                       .30422   R-Bar-Squared                   .30192
 S.E. of Regression              4.7645   F-stat.    F(  1, 302)  132.0471[.000]
 Mean of Dependent Variable  -.098684   S.D. of Dependent Variable      5.7025
 Residual Sum of Squares        6855.5   Equation Log-likelihood      -904.9561
 Akaike Info. Criterion      -906.9561   Schwarz Bayesian Criterion   -910.6731
 DW-statistic                    1.7160
*******************************************************************************


                              Diagnostic Tests
*******************************************************************************
*    Test Statistics  *        LM Version        *         F Version          *
*******************************************************************************
*                     *                          *                            *
* A:Serial Correlation*CHSQ(  12)=  36.2728[.000]*F(  12, 290)=   3.2742[.000]*
*                     *                          *                            *
* B:Functional Form   *CHSQ(   1)=   1.3857[.239]*F(   1, 301)=   1.3783[.241]*
*                     *                          *                            *
* C:Normality         *CHSQ(   2)=   1464.8[.000]*       Not applicable       *
*                     *                          *                            *
* D:Heteroscedasticity*CHSQ(   1)=   4.4829[.034]*F(   1, 302)=   4.5201[.034]*
*******************************************************************************
   A:Lagrange multiplier test of residual serial correlation
   B:Ramsey's RESET test using the square of the fitted values
   C:Based on a test of skewness and kurtosis of residuals
   D:Based on the regression of squared residuals on squared fitted values
```

And then:

```
                      Ordinary Least Squares Estimation
   Based on Newey-West adjusted S.E.'s Bartlett weights, truncation lag= 12
*******************************************************************************
 Dependent variable is DDJSE
 304 observations used for estimation from 1973M4  to 1998M7
*******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 C                           .46821             .21258             2.2025[.028]
 DJSE(-1)                  -.67319            .065441           -10.2870[.000]
*******************************************************************************
```

- Inference:
  - $\rho < 1$
  - Stationary

2.5 Perron Test: Structural Breaks

- Perron (1989): trend-stationary series with a structural break,
  - that is a series which has undergone a permanent change either in slope or intercept at some point during the time period being tested for stationarity,
  - would be considered as a persistent innovation in a series with stochastic trend by the usual ADF-statistic
  - See also Holden and Perman (1994).
  - Thus the test would show nonstationarity, where in fact stationarity applies.

- Alternative responses:
  - Timing of break is known
  - Timing of break is not known

1. Where the time point at which the break occurs in the series is known:
   (a) Include dummy variables into the deterministic component of the relevant DF or ADF statistic, to correct for the intercept of slope change of the series.
      i. Critical values for the unit root tests are found in Perron (1989, 1990).

(b) Conduct a further set of tests proposed by Perron (1994):

i. Innovational outlier model: where structural breaks do not have an instantaneous effect:

$$\begin{aligned} y_t &= \mu + \beta t + \theta DU_t + \gamma DT_t + \delta DTB_t + \alpha y_{t-1} \\ &\quad + \sum \alpha_i \Delta y_{t-i} + e_t \\ H_0 &: \alpha = 1 \\ DU_t &= 1, DT_t = t - T_b if\ t > T_b\ ,\ 0\ otherwise \\ DTB_t &= 1\ if\ t = T_b + 1\ ,\ 0\ otherwise \end{aligned}$$

where T_b denotes the time point of structural break.

ii. Additive outlier model where the effect of the break is instantaneous:

$$\begin{aligned} y_t^* &= \alpha y_{t-1}^* + \sum d_j DTB_{t-j} + \sum \alpha_i \Delta y_{t-i}^* + e_t \\ H_0 &: \alpha = 1 \\ y_t^* &= y_t - \mu - \beta t - \gamma DT_t \\ DTB_t &= 1 \; if \; t = T_b + 1 \, , \; 0 \; otherwise \end{aligned}$$

iii. Critical values are to be found in Perron (1994: Tables 4.2 to 4.5).

2. The time point for a structural break is unknown.

   $\Longrightarrow$ Resort to recursive, rolling or sequential tests for unit roots:

   (a) The usual DF or ADF statistics are computed over various sub-samples of the data (not entirely reliable).

(b) Recursive tests: the usual DF or ADF statistics are computed over subsamples $t = 1, ..., k$, for $k = k_1, ..., T$, where k_1 is a start-up value, T full sample size. The minimum value of $\tau_T\left(k/T\right)$ obtained from all subsamples is chosen and compared to the critical values listed in Banerjee, Lumsdaine and Stock (1992) Table 1, to test for the null of a unit root.

(c) Rolling tests: proceed as for recursive tests, except that the sub-samples are a constant fraction of the total sample, and the fraction over which estimation takes place "rolls" through the sample.

(d) Sequential tests: use an adaptation of the ADF test:

$$\Delta y_t = \mu + \beta_1 t + \beta_2 D + \vartheta_1 y_{t-i} + \sum_{i=1}^{p} \theta_i \Delta y_{t-i} + u_t \tag{2}$$

which allows for the shift in trend model where:

$$\begin{aligned} D &= t \ for \ t > k \\ D &= 0 \ for \ t \leq k \end{aligned} \tag{3}$$

and the shift in mean model where:

$$\begin{aligned} D &= 1 \ for \ t > k \\ D &= 0 \ for \ t \leq k \end{aligned} \tag{4}$$

But k is unknown;

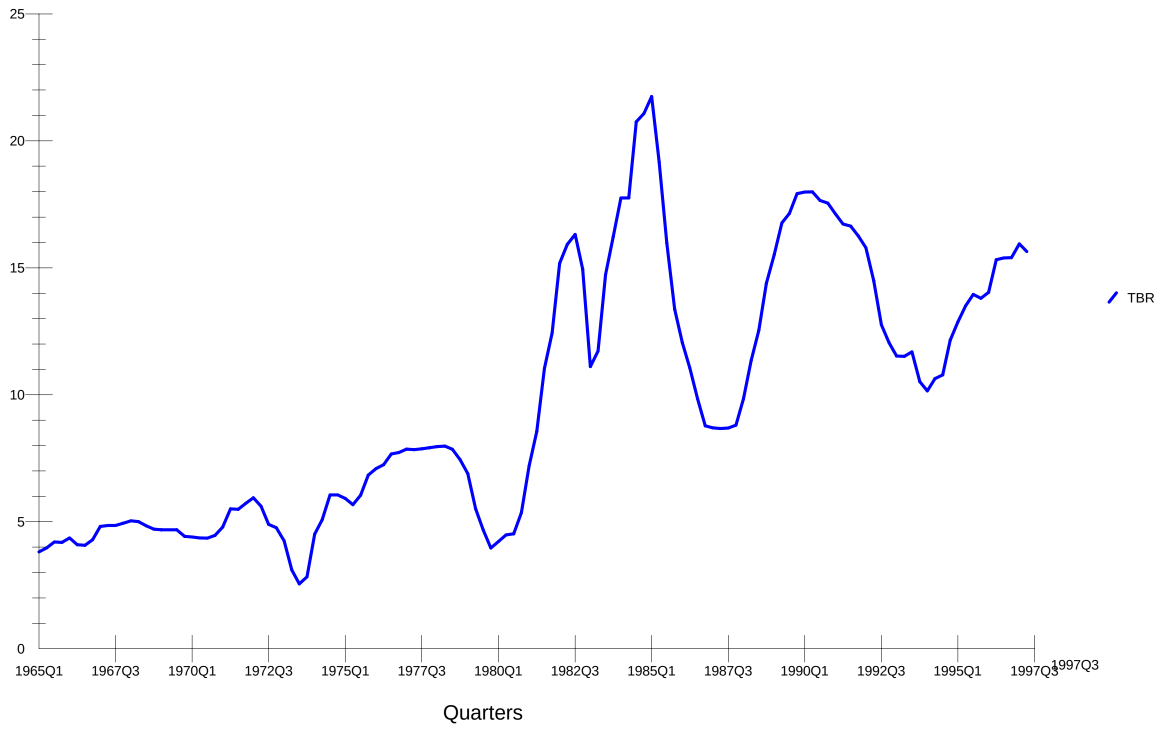
$\Longrightarrow$ Banerjee, Lumsdaine and Stock (1992) suggest searching through $0.15T \leqslant k \leqslant T - 0.15T$.

$\Longrightarrow$ The minimum value of $\tau_{\mathcal{T}}(k/T)$ from the search is compared to the critical values listed in Banerjee, Lumsdaine and Stock (1992) Table 2, to test for the null of a unit root.

$\Longrightarrow$ We may also use the F-test of $H_0 : \beta_2 = \vartheta_1 = 0$, and compare the largest sequentially calculated F-value to the critical values listed in Banerjee, Lumsdaine and Stock (1992) Table 2.

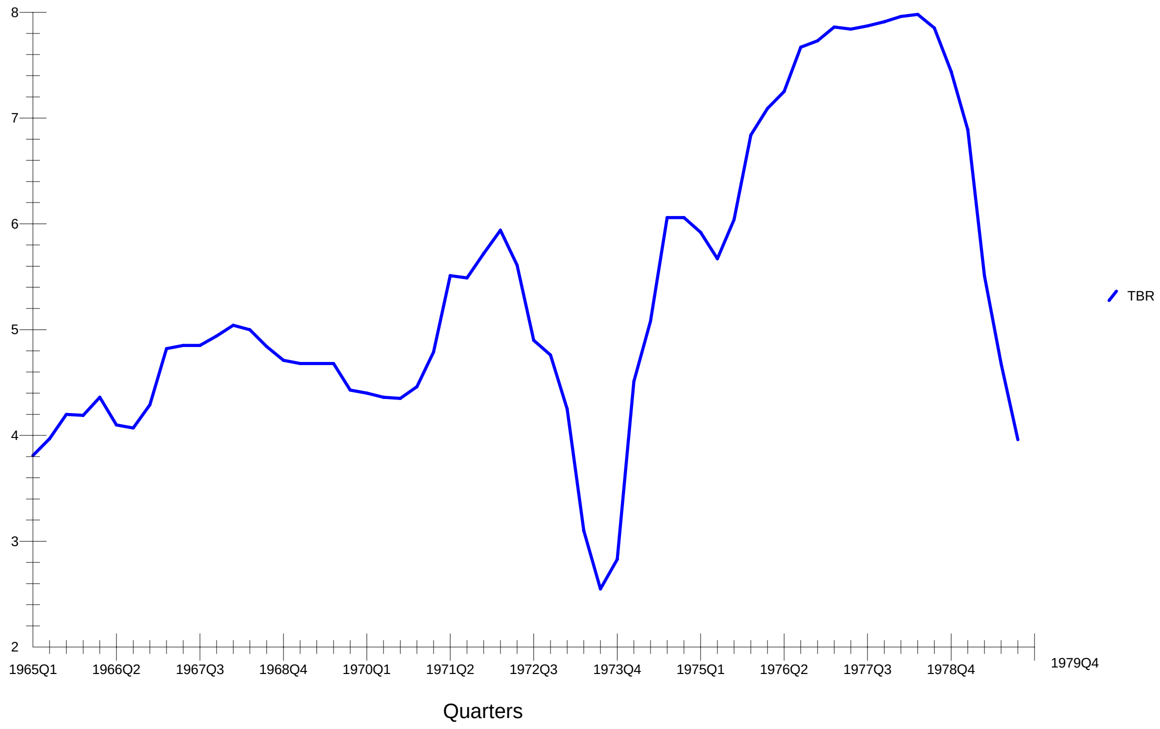
(e) Sequential test procedures can also be applied to the Perron (1994) innovational outlier and additive outlier models.

- EXAMPLE: South African Treasury Bill Rate, 1965Q1-1997Q3

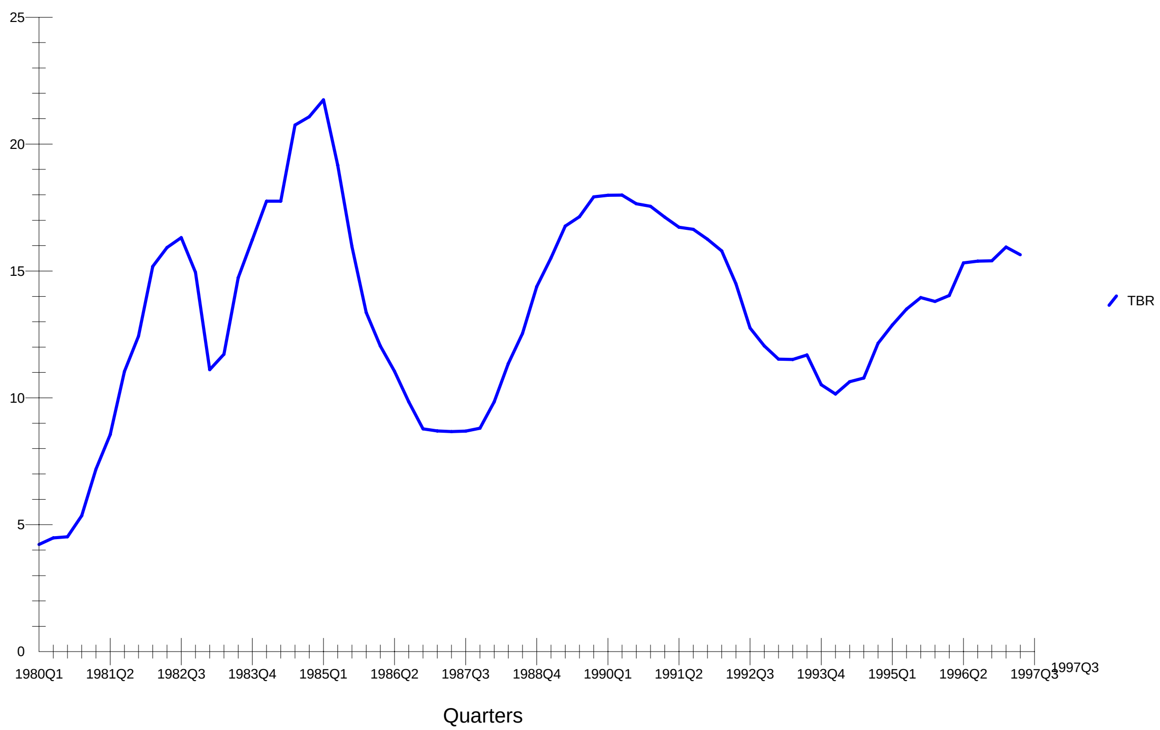


- Plausibly:
  - Structural Break, 1980Q1.
  - Monetary policy regime change: shift to market determined interest rates.
  - Stationary before & after?
  - Shift to higher mean & variance?
- Note: association with clear event

- Since:


8
7
6
5
4
3
2
1965Q1
1966Q2
1967Q3
1968Q4
1970Q1
1971Q2
1972Q3
1973Q4
1975Q1
1976Q2
1977Q3
1978Q4
1979Q4
Quarters
TBR

- And:



25
20
15
10
5
0
1980Q1
1981Q2
1982Q3
1983Q4
1985Q1
1986Q2
1987Q3
1988Q4
1990Q1
1991Q2
1992Q3
1993Q4
1995Q1
1996Q2
1997Q3
1997Q3
Quarters
TBR

- Also:

Unit root tests for variable TBR
The Dickey-Fuller regressions include an intercept and a linear trend

117 observations used in the estimation of all ADF regressions.
Sample period from 1968Q2 to 1997Q2

	Test Statistic	LL	AIC	SBC	HQC
DF	-1.7190	-169.7626	-172.7626	-176.9058	-174.4447
ADF(1)	-3.4902	-141.0734	-145.0734	-150.5978	-147.3162
ADF(2)	-3.6778	-140.3923	-145.3923	-152.2977	-148.1958
ADF(3)	-3.2445	-140.1532	-146.1532	-154.4397	-149.5174
ADF(4)	-3.5449	-139.0355	-146.0355	-155.7031	-149.9604
ADF(5)	-3.7210	-138.3619	-146.3619	-157.4106	-150.8475
ADF(6)	-2.9935	-137.0868	-146.0868	-158.5166	-151.1331
ADF(7)	-2.8620	-137.0868	-147.0868	-160.8977	-152.6938
ADF(8)	-3.0365	-136.5012	-147.5012	-162.6931	-153.6689
ADF(9)	-3.0680	-136.3027	-148.3027	-164.8757	-155.0311
ADF(10)	-3.6945	-133.2822	-146.2822	-164.2364	-153.5714
ADF(11)	-3.0944	-132.8765	-146.8765	-166.2117	-154.7264
ADF(12)	-2.8695	-132.8674	-147.8674	-168.5837	-156.2780

95% critical value for the augmented Dickey-Fuller statistic = -3.4484
LL = Maximized log-likelihood AIC = Akaike Information Criterion
SBC = Schwarz Bayesian Criterion HQC = Hannan-Quinn Criterion

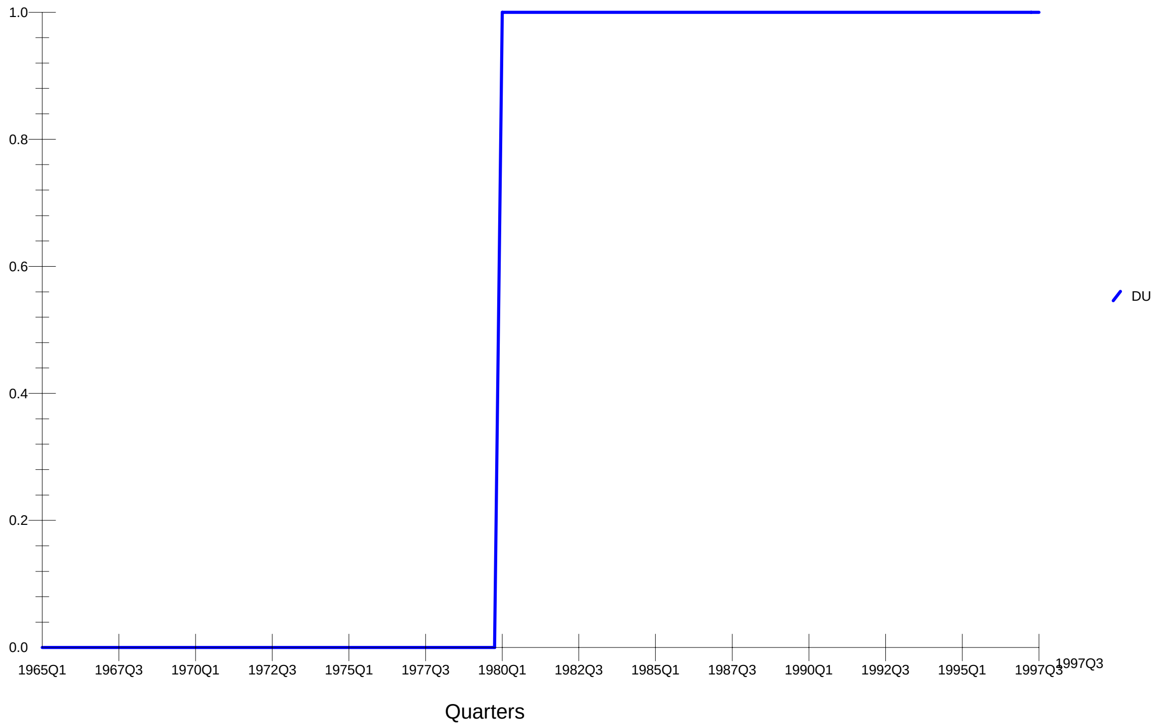
- $\Longrightarrow$ stationary!

- Though:

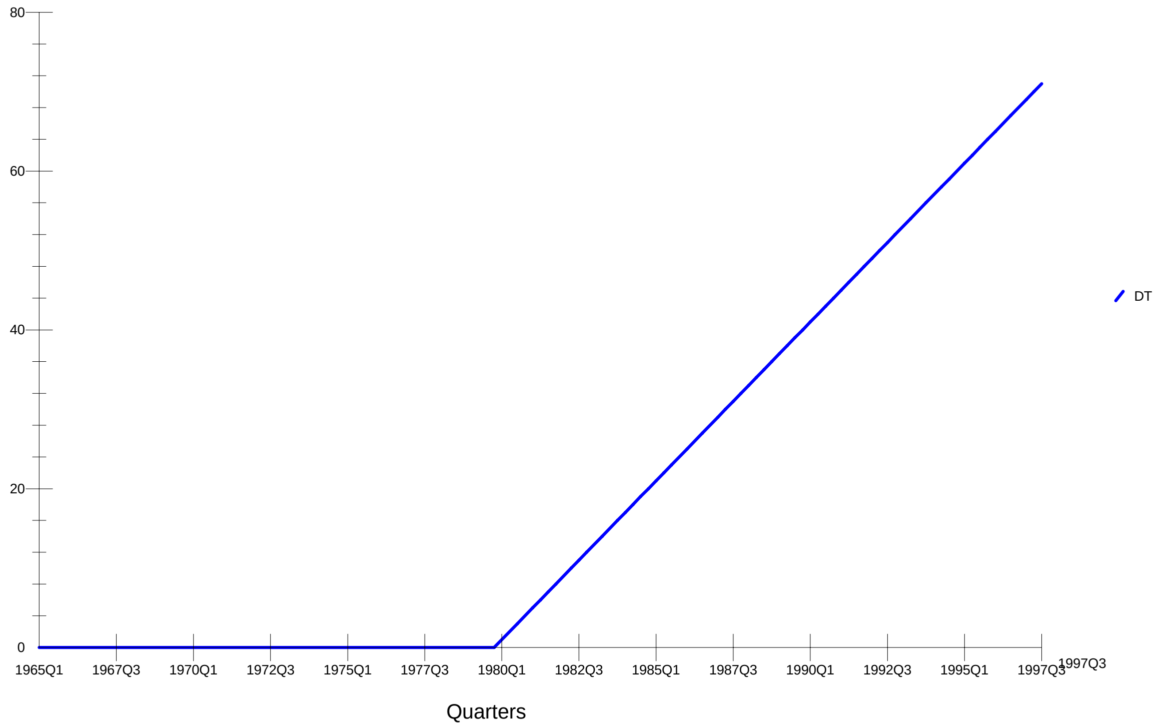
```
                       Unit root tests for variable TBR
      The Dickey-Fuller regressions include an intercept but not a trend
*******************************************************************************
 117 observations used in the estimation of all ADF regressions.
 Sample period from 1968Q2 to 1997Q2
*******************************************************************************
        Test Statistic      LL           AIC           SBC           HQC
 DF          -1.1236     -170.6212     -172.6212     -175.3834     -173.7426
 ADF(1)      -2.2727     -144.4706     -147.4706     -151.6138     -149.1527
 ADF(2)      -2.3118     -144.3593     -148.3593     -153.8837     -150.6022
 ADF(3)      -1.9825     -143.4364     -148.4364     -155.3418     -151.2399
 ADF(4)      -2.0939     -143.0978     -149.0978     -157.3843     -152.4620
 ADF(5)      -2.0989     -143.0607     -150.0607     -159.7283     -153.9856
 ADF(6)      -1.6741     -140.2671     -148.2671     -159.3158     -152.7527
 ADF(7)      -1.5623     -140.0970     -149.0970     -161.5267     -154.1433
 ADF(8)      -1.5986     -140.0019     -150.0019     -163.8128     -155.6089
 ADF(9)      -1.5626     -139.9954     -150.9954     -166.1874     -157.1632
 ADF(10)     -1.7516     -138.8126     -150.8126     -167.3856     -157.5410
 ADF(11)     -1.4953     -136.8455     -149.8455     -167.7996     -157.1347
 ADF(12)     -1.3801     -136.3587     -150.3587     -169.6939     -158.2085
*******************************************************************************
 95% critical value for the augmented Dickey-Fuller statistic =  -2.8861
 LL  = Maximized log-likelihood      AIC = Akaike Information Criterion
 SBC = Schwarz Bayesian Criterion    HQC = Hannan-Quinn Criterion
```

- Countervailing.
- So: Perron Innovational Outlier test: non-instantaneous effect.

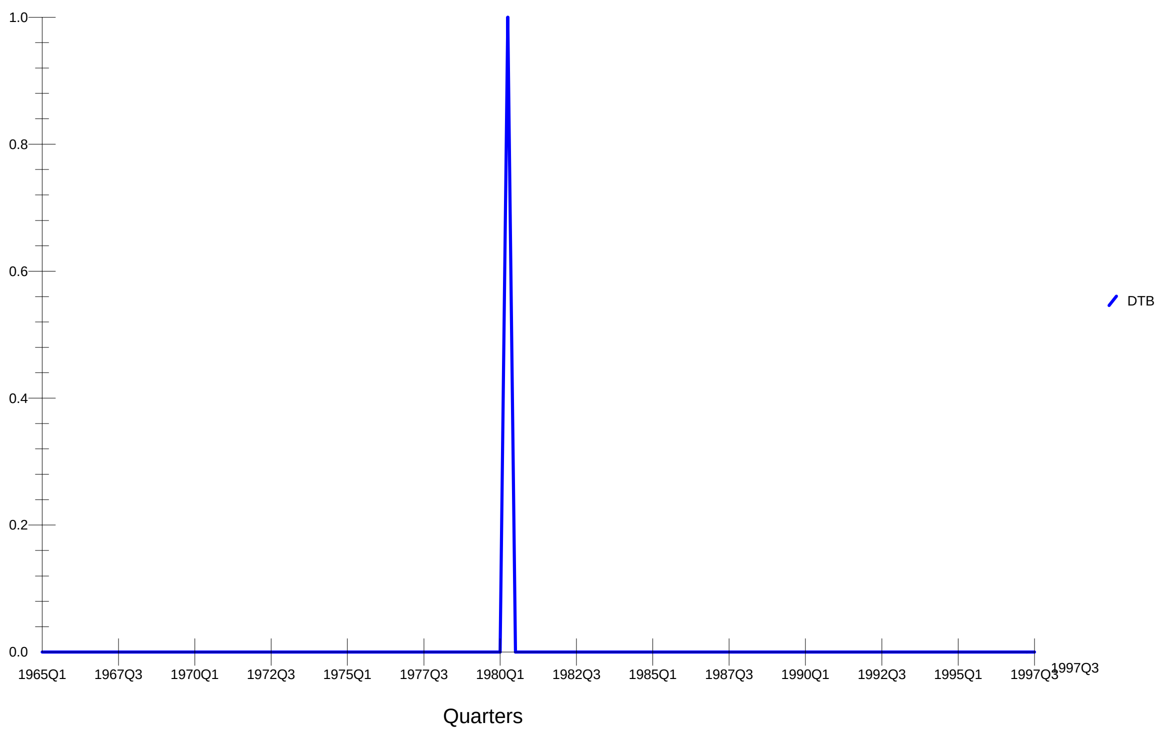
- Construct Dummies: DU:



- DT:


80
60
40
20
0
1965Q1
1967Q3
1970Q1
1972Q3
1975Q1
1977Q3
1980Q1
1982Q3
1985Q1
1987Q3
1990Q1
1992Q3
1995Q1
1997Q3
1997Q3
Quarters
DT

- DTB:



1.0
0.8
0.6
0.4
0.2
0.0
1965Q1
1967Q3
1970Q1
1972Q3
1975Q1
1977Q3
1980Q1
1982Q3
1985Q1
1987Q3
1990Q1
1992Q3
1995Q1
1997Q3
1997Q3
Quarters
DTB

- Then estimate:

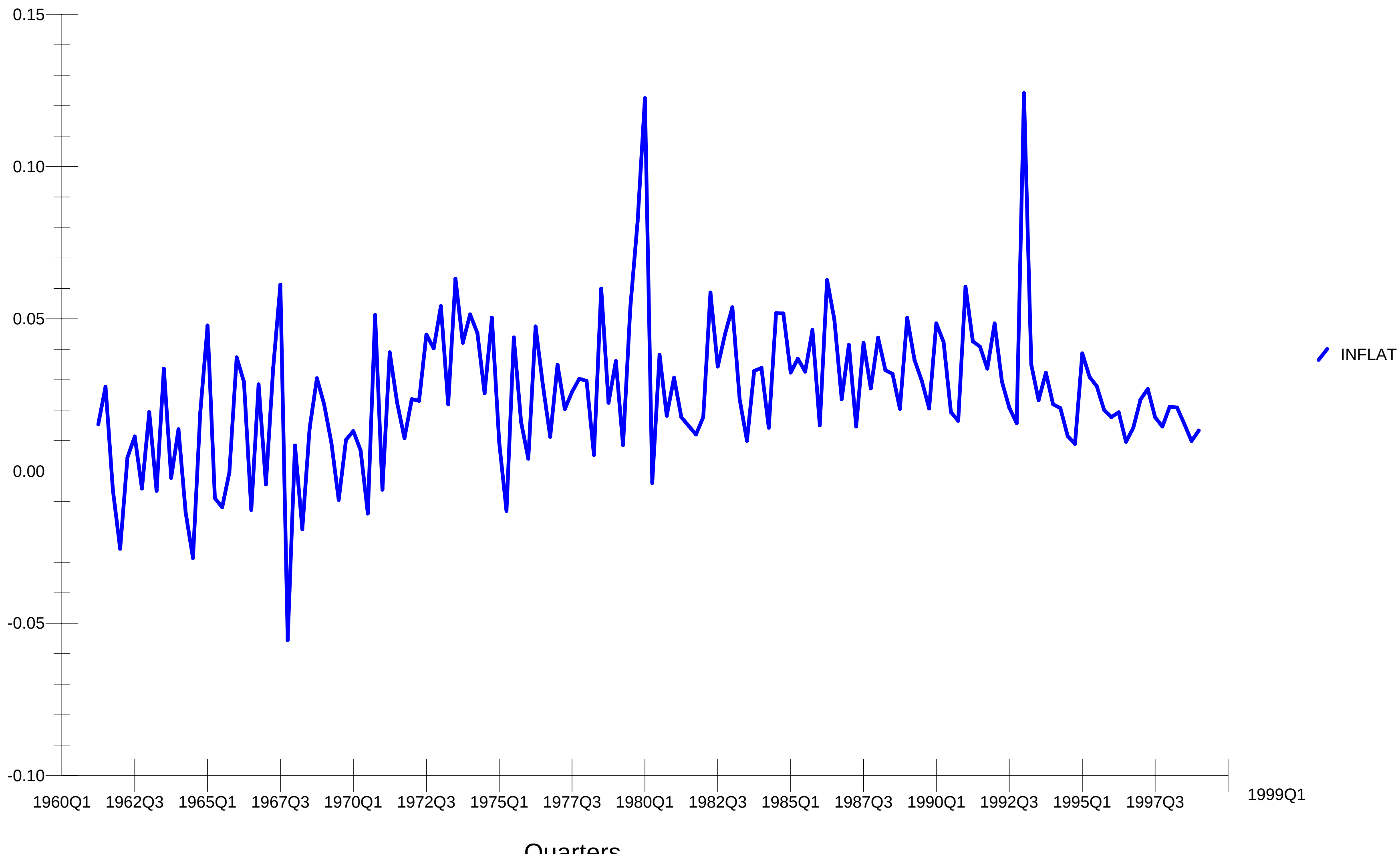
$$\begin{aligned} TBR_t = & \ \mu + \beta t + \theta DU_t + \gamma DT_t + \delta DTB_t \\ & + \alpha TBR_{t-1} + \sum \lambda_i \Delta TBR_{t-i} + e_t \end{aligned}$$

- Gives:

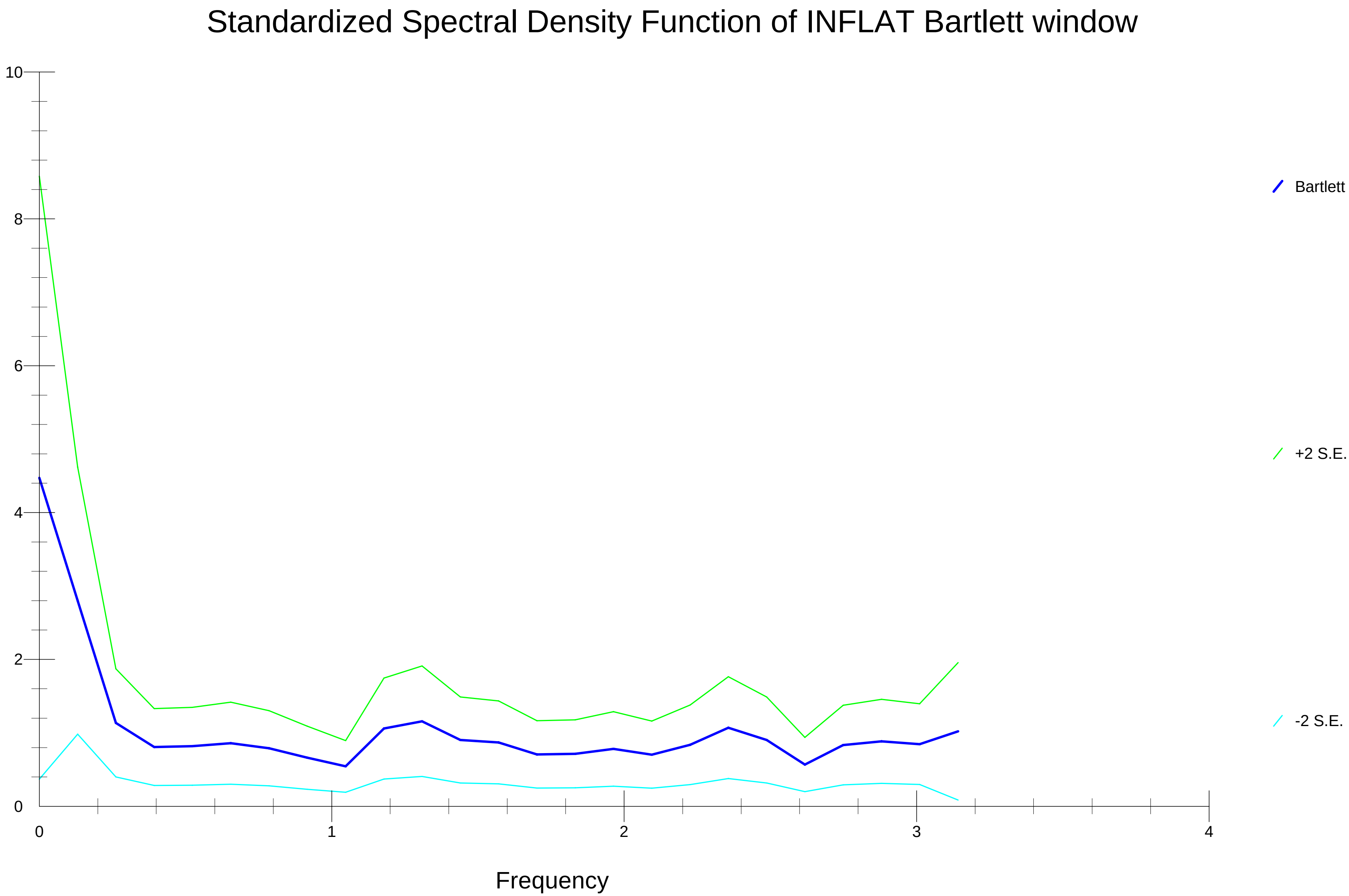
```
                       Ordinary Least Squares Estimation
*******************************************************************************
 Dependent variable is TBR
 125 observations used for estimation from 1966Q2 to 1997Q2
*******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 C                           .52239              .25934            2.0143[.046]
 T                         .0048855            .0067890            .71962[.473]
 DU                          .87503              .30751            2.8455[.005]
 DT                       -.0028986            .0080857           -.35848[.721]
 DTB                        -.82478              .81821           -1.0080[.316]
 TBR(-1)                     .87233             .027755           31.4302[.000]
 DTBR(-1)                    .56373             .085792            6.5709[.000]
 DTBR(-2)                    .12747             .098566            1.2932[.199]
 DTBR(-3)                   -.10879             .099448           -1.0939[.276]
 DTBR(-4)                    .15658             .089820            1.7432[.084]
*******************************************************************************
 R-Squared                     .97794   R-Bar-Squared                    .97621
 S.E. of Regression            .77431   F-stat.    F(  9, 115)  566.4095[.000]
 Mean of Dependent Variable    9.9521   S.D. of Dependent Variable       5.0203
 Residual Sum of Squares      68.9484   Equation Log-likelihood       -140.1826
 Akaike Info. Criterion     -150.1826   Schwarz Bayesian Criterion   -164.3242
 DW-statistic                  2.0774   Durbin's h-statistic     -.45529[.649]
*******************************************************************************


                               Diagnostic Tests
*******************************************************************************
*    Test Statistics  *        LM Version        *         F Version          *
*******************************************************************************
*                     *                          *                            *
* A:Serial Correlation*CHSQ(   4)=   6.2724[.180]*F(   4, 111)=   1.4660[.217]*
*                     *                          *                            *
* B:Functional Form   *CHSQ(   1)=   .62060[.431]*F(   1, 114)=   .56881[.452]*
*                     *                          *                            *
* C:Normality         *CHSQ(   2)= 189.9164[.000]*       Not applicable       *
*                     *                          *                            *
* D:Heteroscedasticity*CHSQ(   1)=   5.7721[.016]*F(   1, 123)=   5.9547[.016]*
*******************************************************************************
   A:Lagrange multiplier test of residual serial correlation
   B:Ramsey's RESET test using the square of the fitted values
   C:Based on a test of skewness and kurtosis of residuals
   D:Based on the regression of squared residuals on squared fitted values
```

- So: $Perron = \frac{0.87233 - 1}{0.027755} = -4.60$
- Critical value: -5.08
- Inference: Nonstationary

- EXAMPLE: South African Inflation: CPI-based 1960Q1-1999Q1



- Given:


Standardized Spectral Density Function of INFLAT Bartlett window
10
8
6
4
2
0
0
1
2
3
4
Frequency
Bartlett
+2 S.E.
-2 S.E.

- Though: ADF's ambiguous: τ_τ:

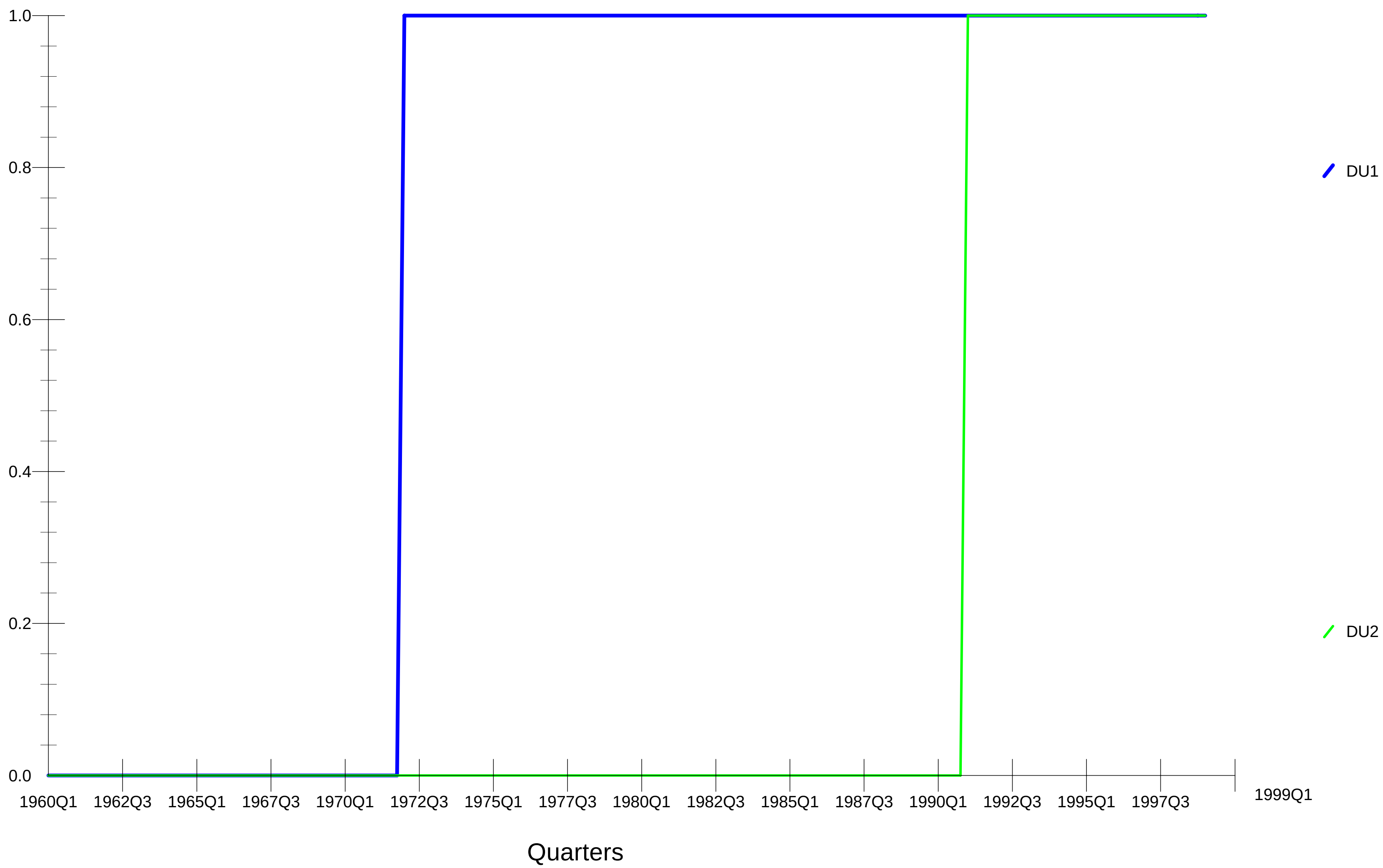
```
                      Unit root tests for variable INFLAT
    The Dickey-Fuller regressions include an intercept and a linear trend
*******************************************************************************
 139 observations used in the estimation of all ADF regressions.
 Sample period from 1964Q3 to 1999Q1
*******************************************************************************
        Test Statistic      LL          AIC          SBC          HQC
 DF        -10.6534      329.4391     326.4391     322.0374     324.6503
 ADF(1)     -7.2104      329.8780     325.8780     320.0091     323.4930
 ADF(2)     -5.5607      330.4227     325.4227     318.0866     322.4415
 ADF(3)     -4.4221      331.2341     325.2341     316.4307     321.6566
 ADF(4)     -3.5486      332.4454     325.4454     315.1748     321.2717
 ADF(5)     -3.0975      332.7694     324.7694     313.0315     319.9995
 ADF(6)     -3.0529      332.8348     323.8348     310.6297     318.4686
 ADF(7)     -2.6553      333.2676     323.2676     308.5952     317.3051
 ADF(8)     -2.3444      333.6057     322.6057     306.4661     316.0470
 ADF(9)     -2.0274      334.1673     322.1673     304.5604     315.0123
 ADF(10)    -1.7713      334.5761     321.5761     302.5020     313.8249
 ADF(11)    -1.9270      335.0131     321.0131     300.4718     312.6656
 ADF(12)    -1.8322      335.0187     320.0187     298.0101     311.0750
*******************************************************************************
 95% critical value for the augmented Dickey-Fuller statistic =  -3.4426
 LL  = Maximized log-likelihood      AIC = Akaike Information Criterion
 SBC = Schwarz Bayesian Criterion    HQC = Hannan-Quinn Criterion
```

- And: τ_μ:

```
                      Unit root tests for variable INFLAT
      The Dickey-Fuller regressions include an intercept but not a trend
*******************************************************************************
 139 observations used in the estimation of all ADF regressions.
 Sample period from 1964Q3 to 1999Q1
*******************************************************************************
        Test Statistic      LL           AIC           SBC           HQC
 DF        -10.3630      327.4594      325.4594      322.5249      324.2669
 ADF(1)     -6.9597      328.3362      325.3362      320.9345      323.5475
 ADF(2)     -5.3660      329.2932      325.2932      319.4242      322.9082
 ADF(3)     -4.2931      330.4726      325.4726      318.1364      322.4914
 ADF(4)     -3.5166      332.0239      326.0239      317.2205      322.4464
 ADF(5)     -3.1263      332.4859      325.4859      315.2152      321.3121
 ADF(6)     -3.0763      332.5073      324.5073      312.7694      319.7374
 ADF(7)     -2.7632      333.0799      324.0799      310.8748      318.7137
 ADF(8)     -2.5354      333.5087      323.5087      308.8364      317.5463
 ADF(9)     -2.3427      334.1459      323.1459      307.0063      316.5872
 ADF(10)    -2.2118      334.5761      322.5761      304.9693      315.4212
 ADF(11)    -2.2982      334.9911      321.9911      302.9170      314.2399
 ADF(12)    -2.2474      335.0026      321.0026      300.4613      312.6552
*******************************************************************************
 95% critical value for the augmented Dickey-Fuller statistic =  -2.8822
 LL  = Maximized log-likelihood      AIC = Akaike Information Criterion
 SBC = Schwarz Bayesian Criterion    HQC = Hannan-Quinn Criterion
```

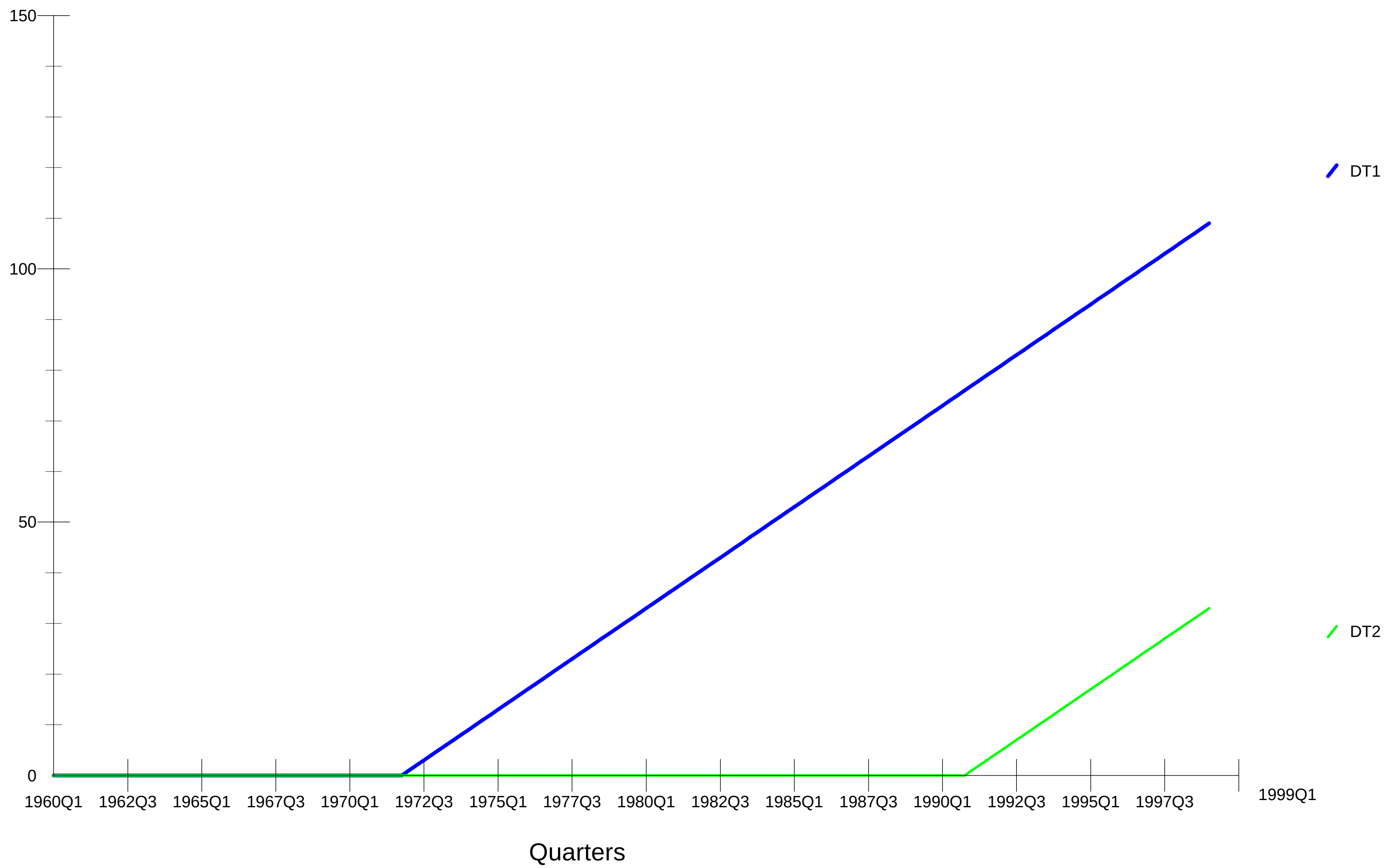
- Hypothesize two structural breaks:
  - 1972Q1: first oil crisis
  - 1991Q1: Stals monetary policy regime shift
- Perron Innovational Outlier test: non-instantaneous effect.
- Two sets of dummies.

- So: DU1, DU2:

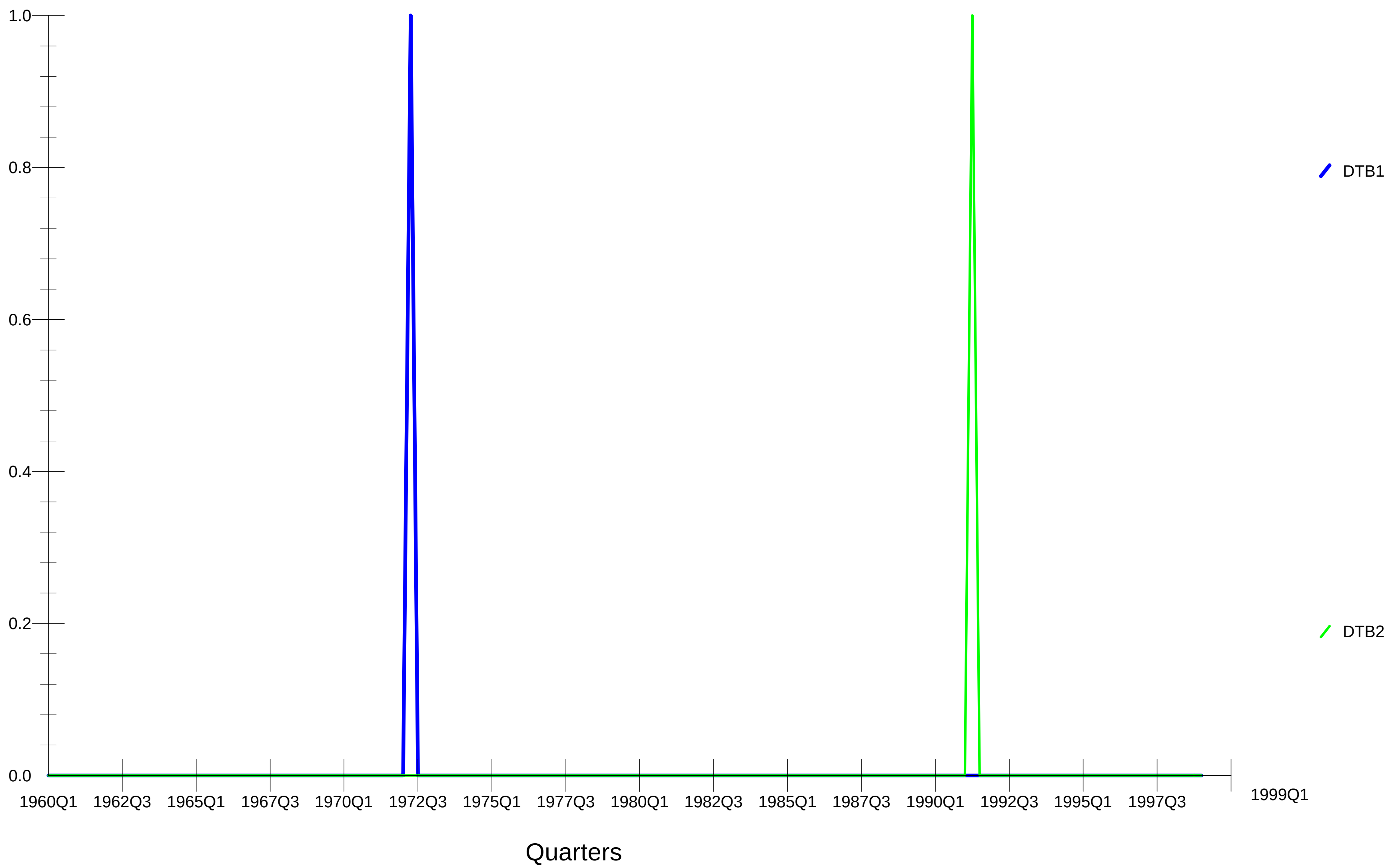


1.0
0.8
0.6
0.4
0.2
0.0
1960Q1
1962Q3
1965Q1
1967Q3
1970Q1
1972Q3
1975Q1
1977Q3
1980Q1
1982Q3
1985Q1
1987Q3
1990Q1
1992Q3
1995Q1
1997Q3
1999Q1
Quarters
DU1
DU2

- And: DT1, DT2:


150
100
50
0
1960Q1
1962Q3
1965Q1
1967Q3
1970Q1
1972Q3
1975Q1
1977Q3
1980Q1
1982Q3
1985Q1
1987Q3
1990Q1
1992Q3
1995Q1
1997Q3
1999Q1
Quarters
DT1
DT2

- And: DTB1, DTB2:



- Then estimate:

$$
\begin{aligned}
TBR_t = {} & \mu + \beta t + \theta_1 DU1_t + \gamma_1 DT1_t + \delta_1 DTB1_t \\
& + \theta_2 DU2_t + \gamma_2 DT2_t + \delta_2 DTB2_t \\
& + \alpha TBR_{t-1} + \sum \lambda_i \Delta TBR_{t-i} + e_t
\end{aligned}
$$

- To obtain:

```
                       Ordinary Least Squares Estimation
*******************************************************************************
 Dependent variable is INFLAT
 147 observations used for estimation from 1962Q3 to 1999Q1
*******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 C                        .0023699            .0098538            .24051[.810]
 T                        .3585E-3            .3180E-3            1.1271[.262]
 DU1                       .025564            .0093560            2.7323[.007]
 DT1                     -.3379E-3            .3355E-3           -1.0074[.316]
 DTB1                     -.014883             .022071           -.67432[.501]
 DU2                       .015733            .0094839            1.6590[.099]
 DT2                     -.0014404            .4655E-3           -3.0945[.002]
 DTB2                    -.0036555             .022620           -.16161[.872]
 INFLAT(-1)               -.37149               .22318           -1.6646[.098]
 DINFLAT(-1)               .25816               .19529            1.3219[.188]
 DINFLAT(-2)               .16864               .16203            1.0408[.300]
 DINFLAT(-3)               .067140              .12590            .53328[.595]
 DINFLAT(-4)               .029106             .085846            .33905[.735]
*******************************************************************************
 R-Squared                      .24768   R-Bar-Squared                    .18031
 S.E. of Regression            .021133   F-stat.    F( 12, 134)    3.6764[.000]
 Mean of Dependent Variable    .026008   S.D. of Dependent Variable     .023342
 Residual Sum of Squares       .059847   Equation Log-likelihood       365.1864
 Akaike Info. Criterion       352.1864   Schwarz Bayesian Criterion    332.7486
 DW-statistic                   2.0049   Durbin's h-statistic            *NONE*
*******************************************************************************


                               Diagnostic Tests
*******************************************************************************
*    Test Statistics  *        LM Version        *         F Version          *
*******************************************************************************
*                     *                          *                            *
* A:Serial Correlation*CHSQ(   4)=   1.9064[.753]*F(   4, 130)=   .42702[.789]*
*                     *                          *                            *
* B:Functional Form   *CHSQ(   1)=   1.8453[.174]*F(   1, 133)=   1.6908[.196]*
*                     *                          *                            *
* C:Normality         *CHSQ(   2)= 142.2319[.000]*       Not applicable       *
*                     *                          *                            *
* D:Heteroscedasticity*CHSQ(   1)=  .019390[.889]*F(   1, 145)=  .019129[.890]*
*******************************************************************************
   A:Lagrange multiplier test of residual serial correlation
   B:Ramsey's RESET test using the square of the fitted values
   C:Based on a test of skewness and kurtosis of residuals
   D:Based on the regression of squared residuals on squared fitted values
```

- So: $Perron = \frac{-0.37149-1}{0.22318} = -6.15$
- Critical value: -5.08
- Inference: Stationary

2.6 Seasonal Unit Root Tests

- Time series data is usually presented in both seasonally adjusted, and seasonally unadjusted form.
- In general, tests for stationarity should be conducted on seasonally **unadjusted** data, since the filters which remove seasonality from the data distort the underlying properties of the data[1].
- In particular, estimates of ρ in $(A)DF$ tests tend to be biased toward 1, thus tending to find nonstationarity in the data more often than is appropriate.

- Two types of seasonal patterns:

1. Stationary seasonal patterns show seasonal variation in a variable, but there is no persistent change in the seasonal pattern over time.

   $\Longrightarrow$ Incorporate seasonal dummies.

2. Nonstationary seasonal patterns manifest a changing and varying seasonal pattern over time - precluding the use of seasonal dummies.

   $\Longrightarrow$ for stationarity series will require *seasonal differencing*.

   $\Longrightarrow$ seasonal processes may have more than one unit root - up to four in the case of quarterly data, for instance.

- Consider: seasonal difference operator for quarterly data is given by $\Delta_4 y_t = \left(1 - L^4\right) y_t = y_t - y_{t-4}$, where $\left(1 - L^4\right)$ can be factorized into:

$$\begin{aligned} \left(1 - L^4\right) &= (1 - L)\left(1 + L + L^2 + L^3\right) \quad (5) \\ &= (1 - L)(1 + L)\left(1 + L^2\right) \\ &= (1 - L)(1 + L)(1 + iL)(1 - iL) \end{aligned}$$

$\Longrightarrow$ 4 roots, with each root corresponding to a different cycle in the time domain:

- The first root $(1 - L)$ is the unit root we have considered thus far, at what we term the zero frequency.
- The second root $(1 + L)$ corresponds to the two-quarter, half-yearly, frequency.
- The pair of complex conjugate roots $(1 \pm iL)$ correspond to the four quarter, annual, frequency.

- Implication: test for the presence of seasonal unit roots.
- Appropriate tests are outlined by Osborne (1990).
- Note:
  - seasonal differencing involves using $(1-L)$ to difference at the zero frequency, d, and
  - the seasonal filter $\left(1+L+L^2+L^3\right)$ to difference at the seasonal frequency D.
  - Thus a stochastic process y_t is said to be integrated of orders d and D, denoted $I(d,D)$, if the series becomes stationary after first period differencing d times, and seasonal differencing D times.

- Proposal, use:

$$\begin{aligned}(1-L)\left(1-L^4\right)y_t &= \Delta\Delta_4 y_t \\ &= \alpha_1 D_{1t}+\alpha_2 D_{2t}+\alpha_3 D_{3t}+\alpha_4 D_{4t} \\ &\quad +\beta_1 \Delta_4 y_{t-1} \\ &\quad +\beta_2 \Delta y_{t-4}+\sum_{i=1}^{p-1}\theta_i \Delta\Delta_4 y_{t-i}+u_t \\ u_t &\sim IID(0,\sigma^2)\end{aligned}$$

  where D_{jt} is the dummy corresponding to quarter j, in order to test the null of $H_0 : I(1,1)$.

- Where: $\beta_1 < 0$ and $\beta_2 = 0$,

  $\Longrightarrow$ acceptance of $H_a : I(0,1)$,

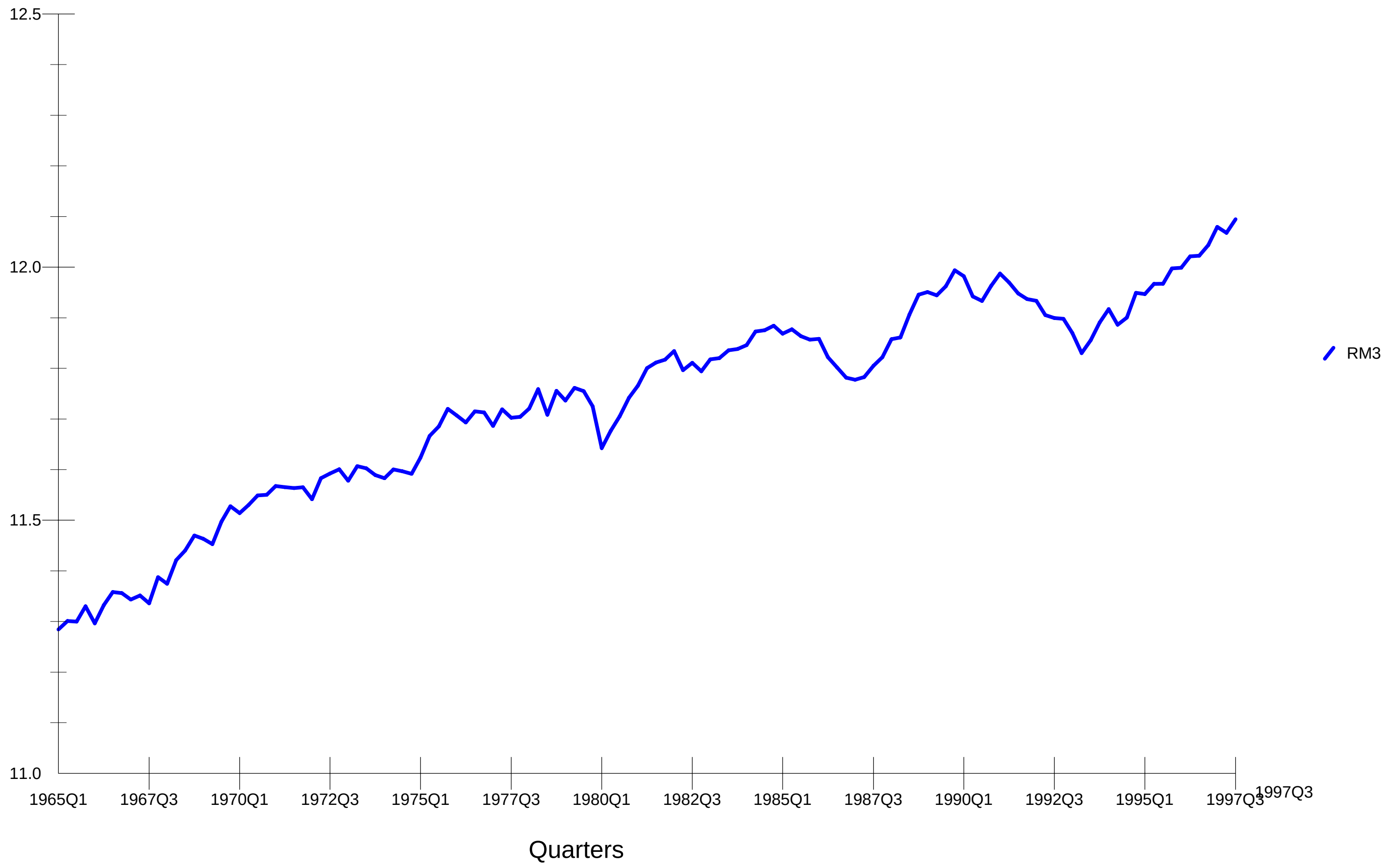
- If $\beta_1 = 0$ and $\beta_2 < 0$,

  $\Longrightarrow$ acceptance of $H_a : I(1,0)$.

- Note that:
  - Seasonal unit roots appear to be relatively rare in macroeconomic data. Osborn (1990) reports only 5 out of 30 series examined as containing a unit root.
  - Instead, most macroeconomic series appear to be $I(1)$ with superimposed deterministic seasonal patters.

- There are good “deep” reasons for why this might be the case:
  * If all three seasonal unit roots are present, then no two quarters will prove to be cointegrated, such that the four quarter series go their separate ways in the long run. The obvious question would be what economic series is likely to generate such a pattern.
- Even where seasonal unit roots are present, the standard ADF test remains valid at the zero frequency, as long as sufficient lags to take account of seasonal terms are included in the test - though some size distortion occurs.

- Example: South African Broad Real Money Balances (LOG-SCALE)



– Estimate:

$$\begin{aligned}\Delta\Delta_4 RM3_t &= \alpha_1 D_{1t} + \alpha_2 D_{2t} + \alpha_3 D_{3t} + \alpha_4 D_{4t} + \beta_1 \Delta_4 RM3_{t-1} \\ &\quad + \beta_2 \Delta RM3_{t-4} + \sum_{i=1}^{p-1} \theta_i \Delta\Delta_4 RM3_{t-i} + u_t\end{aligned}$$

– Where:

$$\begin{aligned}\Delta_4 RM3 &\equiv RM3_t - RM3_{t-4} \\ \Delta\Delta_4 RM3 &\equiv D4RM3_t - D4RM3_{t-1} \\ \Delta RM3 &\equiv RM3_t - RM3_{t-1}\end{aligned}$$

– Where $\beta_1 < 0$ and $\beta_2 = 0$ it follows that $RM3 \sim I(0,1)$,

– Where $\beta_1 = 0$ and $\beta_2 < 0$ it follows that $RM3 \sim I(1,0)$.

– Gives:

```
                       Ordinary Least Squares Estimation
*******************************************************************************
 Dependent variable is DD4RM3
 106 observations used for estimation from 1971Q2 to 1997Q3
*******************************************************************************
 Regressor              Coefficient       Standard Error         T-Ratio[Prob]
 S1                       .0014831            .0049120            .30193[.763]
 S2                       .0055387            .0047949            1.1551[.251]
 S3                       .0061855            .0047968            1.2895[.200]
 S4                        .011754            .0049169            2.3906[.019]
 D4RM3(-1)                -.096301             .076020           -1.2668[.208]
 DRM3(-4)                  -.87751              .14738           -5.9540[.000]
 DD4RM3(-1)                 .12967             .084595            1.5328[.129]
 DD4RM3(-2)                .075719             .086556            .87479[.384]
 DD4RM3(-3)                .081696             .087060            .93838[.350]
 DD4RM3(-4)                 .15958              .10117            1.5774[.118]
*******************************************************************************
 R-Squared                     .45226   R-Bar-Squared                   .40091
 S.E. of Regression           .023839   F-stat.    F(  9,  96)    8.8073[.000]
 Mean of Dependent Variable  .1755E-3   S.D. of Dependent Variable     .030800
 Residual Sum of Squares      .054558   Equation Log-likelihood       250.9045
 Akaike Info. Criterion      240.9045   Schwarz Bayesian Criterion    227.5873
 DW-statistic                  1.9462
*******************************************************************************


                               Diagnostic Tests
*******************************************************************************
*    Test Statistics  *        LM Version        *         F Version          *
*******************************************************************************
*                     *                          *                            *
* A:Serial Correlation*CHSQ(   4)=   4.3241[.364]*F(   4,  92)=   .97814[.423]*
*                     *                          *                            *
* B:Functional Form   *CHSQ(   1)=   .69475[.405]*F(   1,  95)=   .62676[.431]*
*                     *                          *                            *
* C:Normality         *CHSQ(   2)=   2.6982[.259]*       Not applicable       *
*                     *                          *                            *
* D:Heteroscedasticity*CHSQ(   1)=   .24950[.617]*F(   1, 104)=   .24537[.621]*
*******************************************************************************
   A:Lagrange multiplier test of residual serial correlation
   B:Ramsey's RESET test using the square of the fitted values
   C:Based on a test of skewness and kurtosis of residuals
   D:Based on the regression of squared residuals on squared fitted values
```

– Thus: $\beta_1 = \underset{(0.076020)}{-0.096301} = 0$

– And: $\beta_2 = \underset{(0.14738)}{-0.87751} < 0$

$\implies RM3_t \sim I(1, 0)$.

– No seasonal unit root.

2.7 Power and level of unit root tests

- Size distortion of ADF tests:
  - Over-rejection of null: too ready conclusion of stationarity
  - Distribution of ADF-tests diverges strongly from reported DF-distribution when underlying distribution contains MA element.
  - Extends to Phillips-Perron.
  - Schwert (1989), De Jong et al (1992), Agiakoglu and Newbold (1992).

- Power distortions of unit root tests:
  - Under-rejection of null: too ready conclusion of non-stationarity
  - Unit root tests low power against plausible trend stationary alternatives
  - De Jong et al (1992), Agiakoglu and Newbold (1992).

- Choosing the correct testing procedure for unit roots not always straightforward.
- ADF tests with different lag lengths and deterministic components can result in different outcomes with respect to testing the null of non-stationarity.
- These problems compound in small sample contexts.

- Problem:
  - when we apply ADF tests to finite samples, it is strictly speaking inappropriate to apply critical values based on the DF asymptotic distribution.
  - difficulty becomes most severe due to the closeness of finite sample distributions of statistics under trend-stationary and difference-stationary processes that approximate trend-stationarity. In finite samples:
    * Any trend-stationary process can be approximated arbitrarily well by a unit root process.
    * Any unit root process can be approximated by a trend stationary process, especially in small samples.

- In finite samples:
  - Some unit root processes appear to behave like trend-stationary processes,
  - Some trend-stationary processes appear to behave like random walk (nonstationary) processes.
  - Unit root tests are likely to be fooled in either event: either rejecting nonstationarity where nonstationarity should be accepted, or accepting nonstationarity where nonstationarity should be rejected.

- One important consequence of this is that one's inferences concerning the stationarity properties of data should always be based on a range of evidence, of which the formal ADF (and other) tests are merely one component.
- Residual plots, plots of the autocorrelation function, the spectrum, etc., all form part of the evidence required to draw conclusions on the stationarity of data.