

# Applied Time Series Econometrics

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# 1. Overview

1. Introduction
2. Methodological Principles of (Time Series) Modelling
3. Some Reminders
4. Some Core Time Series "Engines," for eg.:
  - (a) White noise
  - (b) Moving averages
  - (c) Random Walks
5. ARCH & GARCH Estimation

6. Stationarity & Spurious Regression
7. Integration & Cointegration
8. Establishing the Univariate Time Series Characteristics of the Data
9. Cointegration in Single Equations
10. Cointegration in Multivariate Systems
11. Panel Data Estimation Long in T

## 2. Methodological Principles

### 2.1 The Standard Background:

1. Objectives of econometric modelling:
  - (a) Understanding structure
  - (b) Understanding response to shocks
  - (c) Forecasting
2. Impossible in complete sense
3. Hence:
  - (a) Sufficient detail
  - (b) Parsimonious

$\Rightarrow$  Tension

#### 4. Hence search for legitimacy:

##### (a) Relevance:

- i. As few variables as possible;
- ii. As many variables as necessary;

##### (b) Conditioning:

- i. Weak exogeneity: no contemporaneous feedback  
A. Estimation & Testing
- ii. Strong exogeneity: no Granger causality  
A. Forecasting
- iii. Super exogeneity: parameter vectors of endogenous and exogenous variables independent (Lucas Critique)  
A. Policy analysis

- (c) Functional Form
- (d) Data Evaluation:
  - i. Time series properties
  - ii. Structural breaks
  - iii. Outliers
  - iv. Changes in distribution
- (e) Estimation
- (f) Evaluation:
  - i. Hypothesis testing: specification diagnostics
  - ii. Model simulation:
    - A. Forecast performance
    - B. Response characteristics

⇒ Need for guidelines of model evaluation.

5. Load: METHOD200.FIT

(a) What model is present in the data?

(b) Why?

(c) Note:

$$Y_t = 1 + u_t$$

$$Z_t = 5 + 0.5Y_t + v_t$$

$$X_t = -3 + 0.7Y_t + 0.3Z_t + n_t$$

Hence:

$$Y_t = 1 + u_t$$

$$Z_t = 5 + 0.5u_t + v_t$$

$$X_t = -0.65 + 0.85u_t + 0.3v_t + n_t$$

And:

$$Y_t = 3 + \frac{1}{0.7}X_t - 0.3Z_t - n_t$$

## 2.2 Model Selection and Validation

### 1. Consistency:

- (a) Inadmissability - logically possible:
  - i. Signs
  - ii. Parameter values
  - iii. Forecasts
- (b) Operating characteristics: consistent with observable equilibrium behaviour

### 2. Significance:

- (a) Economic
- (b) Statistical:
  - i. Nominal significance tests: eg. F-tests, t-tests, Wald-tests.
  - ii. Optimized significance tests: eg.  $\sigma^2$ , AIC, SC, HQ.

### 3. Data Adequacy:

(a) Explanatory power → See Significance

(b) Econometric assumptions:

i. LR, Wald, LM types: likelyhood ratios; asymptotically equivalent.

ii. 3 categories:

A. Conditional mean:

- autocorrelation
- misspecification
- predictive failure
- Chow

B. Conditional variance:

- heteroskedasticity: Ramsey, White.

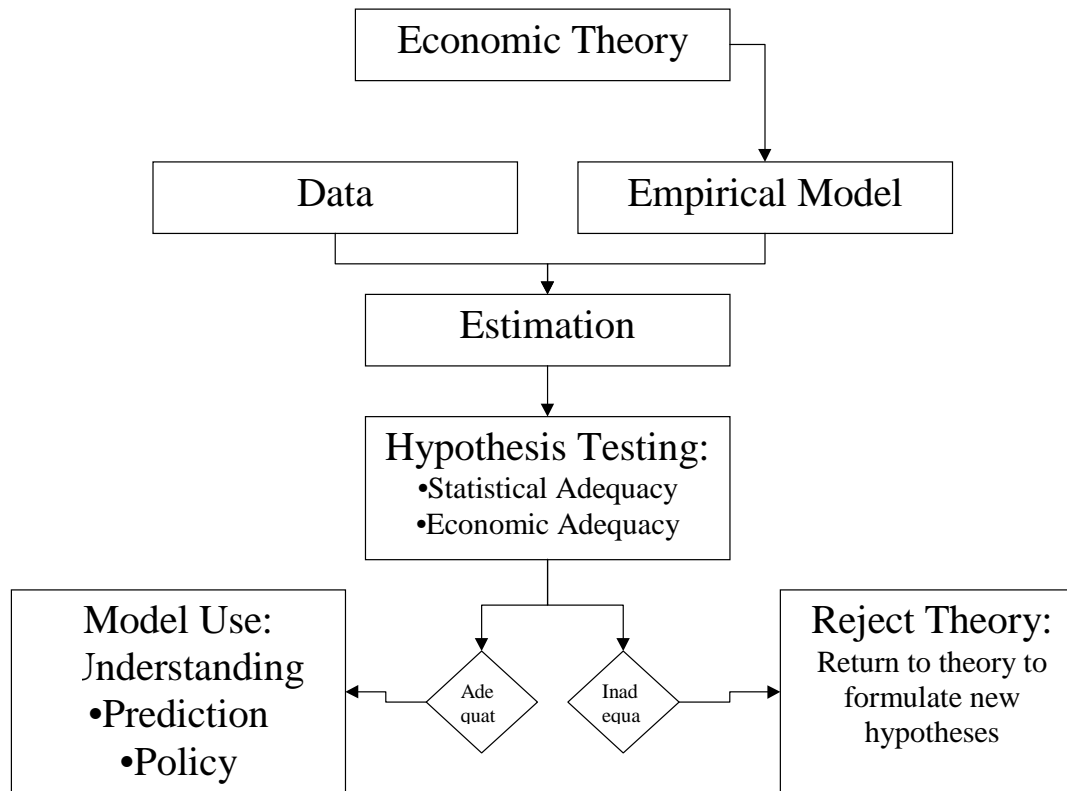
- ARCH

c. Normality

4. Encompassing: rival models should not contain information that the preferred model does not.
5. Sensitivity:
  - (a) Sample size
  - (b) Variable menu
  - (c) Model equations

## 2.3 Rival Methodologies: The Old Stuff

### 1. Cowles Commission or Box-Jenkins:



⇒ Deductive-nomological structure of scientific method

## 2. Principles involved:

- (a) Determination of stock prices
- (b) Participants: Haavelmo\*; Hurwicz; Klein\*; Koopmans\*; Wald; Frisch\*; {Tinbergen\*}.
- (c) Consider the following DGP:

$$Y_t = \alpha X_t + \theta W_t + \varepsilon_{1,t} \quad (1)$$

$$X_t = \beta Y_t + \gamma Z_t + \varepsilon_{2,t} \quad (2)$$

(d) How to model?

...the first thing to be done in any particular application is to give a correct economic analysis of the relation to be investigated ... - Two things should be carefully kept in mind: first, the necessity to know exactly what relation one is really interested in and second to know what factors enter into this relation. Tinbergen (1951:207).

... it is very useful to make a distinction between the simultaneous variables which have to be explained by the set of equations and the so-called "predetermined" variables which may also be found in the equations. Predetermined variables may be either data or lagged values of the the simultaneous variables. Tinbergen (1951:201).

⇒ Principle 1: Appropriate zero restrictions:  
identification.

⇒ Principle 2: Causal structure is essentially  
a priori. Endogenous  $(Y_t, X_t)$ , exogenous  $(W_t,$   
 $Z_t)$  variables known.

⇒ Principle 3: Theory = King.

What should be the mathematical form of the relation? ... A very general mathematical theorem is of advantage in all these cases: namely, that within a small range of variation, nearly every function can be approximated by a linear one. Tinbergen (1951:19-20).

⇒ Principle 4: Linearization.

It is often maintained that the method of multiple correlation is too rigid since it assumes the regression coefficients to be constant. It is very easy, however, to replace those coefficients by functions of other variables which means that they will be variable. ... On the other hand it may be observed that experience supplies us with cases of a remarkable constancy of regression coefficients. Tinbergen (1951:205).

⇒ Principle 5: Relations invariant in time:  $\alpha, \beta, \gamma, \theta$ , invariant in  $t$ .

⇒ Principle 6: Parameters structurally invariant: not allowed to be systematically related to movements in  $(Y_t, X_t, W_t, Z_t)$ , though in  $(M_t, N_t, P_t, Q_t)$  is permissible in part.

The statistician, when testing a relation set up by economic theory, may draw ... only negative [conclusions]. ... If, namely, the correlation coefficient turns out to be too low ... the conclusion must be that the relation is incompatible with the facts. In other words, the theory is incorrect, or at least incomplete. The positive conclusions which can be drawn do not appear when the theoretical relation is checked: it cannot be proved that a theory is correct. The correlation may be high: other theories, however, may also lead to high correlation ... If we want to draw positive conclusions, additional information is necessary ... Tinbergen (1951:84-5).

⇒ Principle 7: The model cannot be tested against “rival” models.

⇒ Principle 8: The model is tested against “nature” by means of fit:  $R^2$ , t-ratios, F-tests.

(e) Evolution:

- i. 24 equation model of Dutch economy - Tinbergen 1936
- ii. Klein's Model 1: 6 equation model of US economy - Klein 1950
- iii. 20 equation model of US economy - Klein & Goldberger 1956
- iv. 160 equation Brookings model of US economy - Duesenberry et al 1965
- v. 200 equation Brookings MARK II model of US economy - McCarthy 1972
- vi. And so on ....

(f) Except:

On pessimistic days I doubt that economists have learned anything from the mountains of computer print-outs that fill their offices. On especially pessimistic days, I doubt they ever will. Leamer (1983:325)

Why?

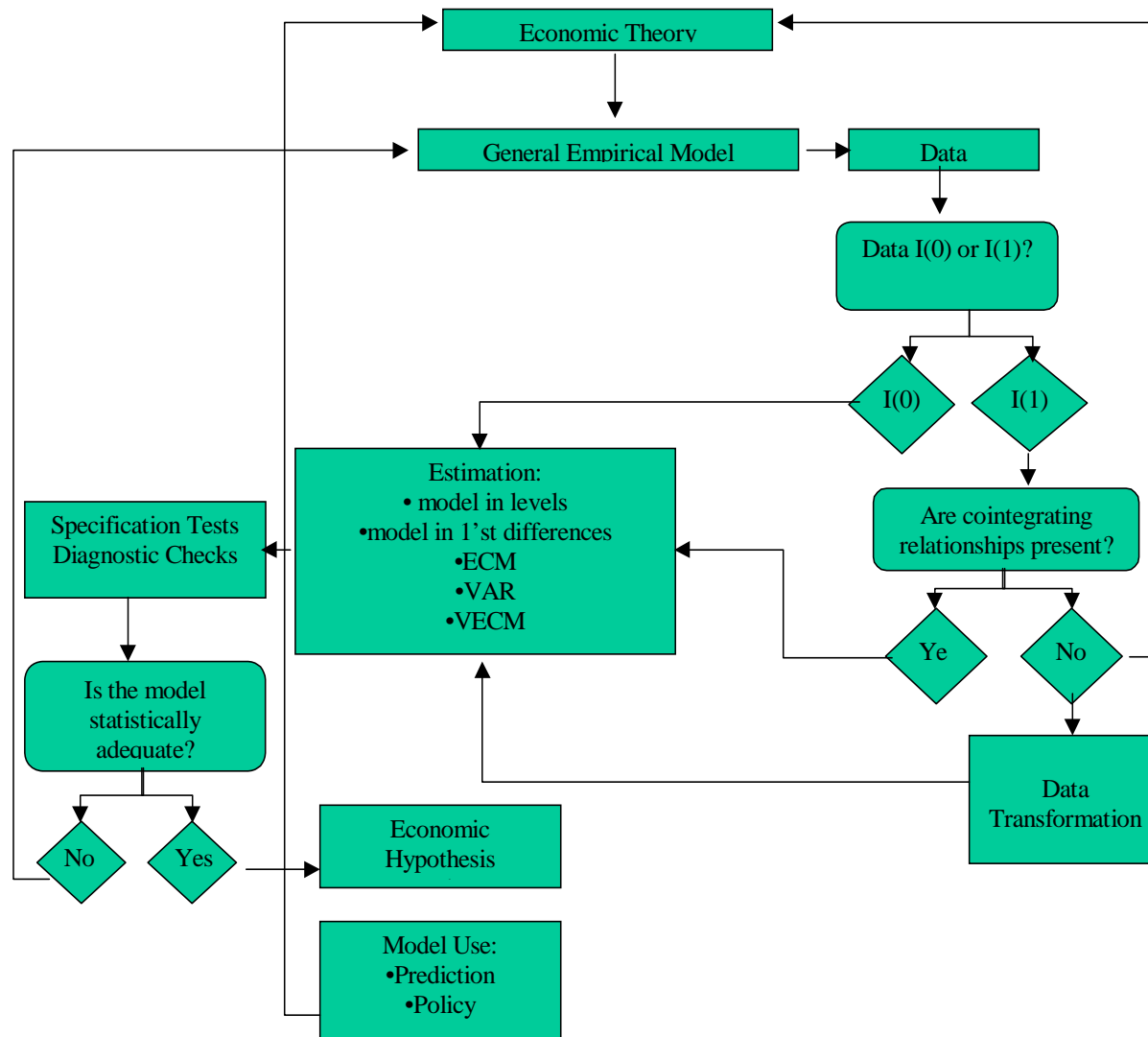
→ Data Mining

→ Standard goodness of fit statistics spurious.

→ Poor forecasting ability of models: big macro models trounced by simple VAR & time series models.

## 2.4 Rival Methodologies: The New Stuff

### 1. LSE Methodology:



⇒ Principle 1: “test, test, test”.

⇒ Principle 2: Data congruence.

⇒ Principle 3: *Interaction* between theory and data.

## 2. Criteria for evaluation:

- (a) Model is coherent with available information
- (b) Model is at least as good as alternative models

⇒ Principle 4: Congruence.

⇒ Principle 5: Encompassing.

## 3. How do we get there? Modelling Strategy

- (a) Specific-to-general
- (b) General-to-specific

## 2.4.1 Congruence

1. With a priori **theory**.

(a) Economics & economic theory

(b) Contrast:

i. Data driven modelling: VAR's

ii. Theory driven modelling:

A. Structural type: Hansen & Sargent

B. Calibrated real business cycle type: Kyd-  
land & Prescott

(c) LSE approach:

- i. Congruence with economic theory desirable
- ii. Congruence with economic theory not necessary:
  - A. New theories, learning relies on observation: Darwin
  - B. Economic theory does not have veracity a priori

- iii. Low-level theory (Hendry) important:
  - A. Which variables to include
  - B. Functional forms that are likely

- iv. High-level (Hendry) theory is not:
  - A. Tight & specific forms
  - B. Why?
    - Presumes statistical test of theory
    - Rarely conclusive: empirical model yields invalid inference because noncongruent with sample information; empirical model not reliable representation of theory; observables do not correspond to economic theory;

2. Load: METHOD200AR.FIT

(a) Estimate:

$$y_t = c + \delta x_t + \varepsilon_t$$

(b) Assess the model

(c) True model given by:

$$\begin{aligned}y_t &= 0.3y_{t-1} + 0.2z_t + 0.4z_{t-1} + u_t \\z_t &= 0.5z_{t-1} + v_t\end{aligned}$$

(d) Hence:

$$y_t = 0.86z_t$$

(e) Under OLS:

$$y_t = 0.47z_t$$

3. With **sample information**: necessary for statistical inference to be valid.

(a) **Relative past sample information**: a model not congruent with past sample information (lagged values of variables), will have serially correlated errors, and in general will lead to invalid & misleading inferences.

Consider:

$$y_t = \delta z_t + u_t \quad (3)$$

where the true association is given by:

$$y_t = \alpha y_{t-1} + \beta z_t + \gamma z_{t-1} + \varepsilon_t$$

$$z_t = \lambda z_{t-1} + \nu_t$$

$$|\alpha| < 1, |\lambda| < 1$$

then in the long run, where:

$$\begin{aligned} y_t &= y^*, z_t = z^*, \forall t \\ \text{"true"} \frac{\partial y_t}{\partial z_t} &= \frac{\beta + \gamma}{1 - \alpha} \\ \widehat{\delta}_{OLS} &= \frac{\beta + \gamma\lambda}{1 - \alpha\lambda} \end{aligned}$$

such that:

$$\begin{aligned} \frac{\partial y_t}{\partial z_t} &= \widehat{\delta}_{OLS} \\ &\text{iff} \\ \gamma &= -\alpha\beta \end{aligned}$$

in which case the ADL model takes the form:

$$\begin{aligned} y_t &= \beta z_t + \varepsilon_t \\ \varepsilon_t &= \alpha \varepsilon_{t-1} + \nu_t \end{aligned}$$

It follows that:

- Testing for serial correlation in errors
- Then correcting by means of the standard techniques for AR
- Will be appropriate iff  $\gamma = -\alpha\beta$  a (very) restrictive condition

- (b) Relative present sample information: a model with errors that are not homoscedastic wrt current values of explanatory variables  $E(u_t | \mathbf{x}_t) \neq 0$
- i. Does not fully exploit all available information
  - ii. Not congruent
  - iii. Reasons:
    - A. Omitted variables
    - B. Conditioning on variables not weakly exogenous
  - iv. Tests:
    - A. Homoscedasticity
    - B. Omitted variables
    - C. Weak exogeneity
    - D. Skewness & Kurtosis

- (c) Relative future sample information:
  - i. Standard tests of:
    - A. predictive failure
    - B. parameter constancy
    - C. eg. variance type parameter constancy; Chow; recursive.
  - ii. Lucas critique:
    - A. where conditioning variables are weakly exogenous
    - B. parameters invariant to process generating the conditioning variables
    - C. A. + B. = superexogeneity
    - D. Lucas critique benign.

4. Measurement system: what is being measured & how is crucial:
  - (a) For functional form
  - (b) For variable transformation
  - (c) For eg.:
    - i. Aggregate consumption:  $\rightarrow$  ln-transform, since  $>0$ , strictly.
    - ii. Unemployment rate:  $\rightarrow$  ln- or logit-transform, or avoid fitting data out of  $[0,1]$  range.
    - iii. Stationarity, integratedness, seasonality, ...

## 5. Rival Models:

- (a) Usually exist many rivals
- (b) More than one may be data-congruent

⇒ How choose?

... to have a model that is congruent ... is a necessary condition .... but it is not sufficient for it to be a dominant or encompassing model. An encompassing model is one which can account for the previous empirical findings that were thought to be relevant and adequate for the explanation of the variables and parameters of interest and can explain the features of rival models being considered currently. The encompassing model renders other models inferentially redundant and so is a dominant model. Mizon (1995:120).

(c) Test by:

i. Non-nested hypothesis testing. Requires:

A. Embedding model (test statistic size & power)

B. Problem:  $M_1 \varepsilon M_2, M_2 \varepsilon M_3 \not\Rightarrow M_1 \varepsilon M_3$  because embedding model may differ

ii. Hence: consider the embedding model  $M_i \subset M_c \forall i$

A. Then if  $M_1 \varepsilon_p M_c, M_1 \varepsilon_p M_j \forall j$

$\Rightarrow$  parsimonious encompassing: acceptable “reduction” of congruent embedding model

6. Encompassing:

(a) parsimonious encompassing

(b) encompasses new information: new theory

7. Specific-to-general:

(a) Model specific & simple: direct from theory

(b) Advantages:

- i. avoids unnecessary generality (complexity) and hence loss of efficiency
- ii. avoids tuning model to sample data
- iii. theory-link aids in the interpretation

(c) Disadvantages:

- i. no guarantee that model statistically well specified; i.e. no guarantee that economic theory has empirical veracity
- ii. always possible to find more than one congruent model → evaluation

8. Hence: General-to-specific